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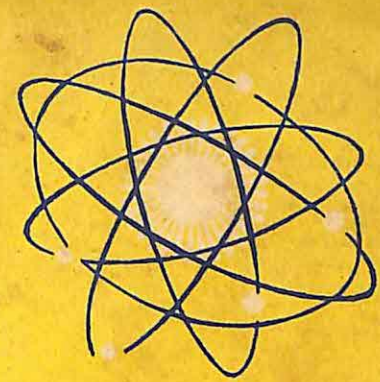
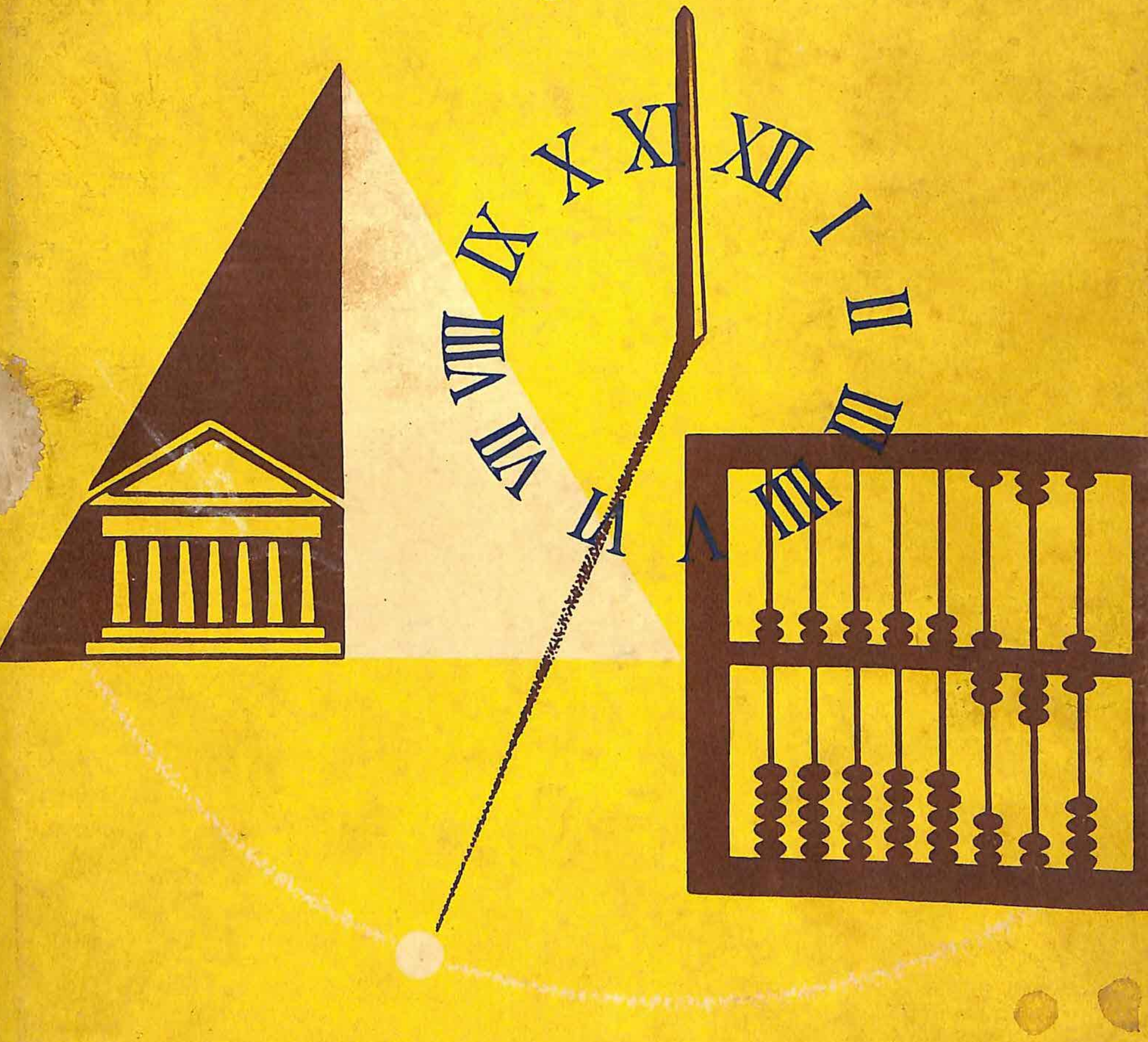
THE STORY OF MATHEMATICS



H Alan Shaw & Keri Fuge



This book continues our popular series of informative books for children in the 10 to 14 age group.

It tells the story of mathematics, stressing how it developed naturally with man's need to count, add and subtract, build and irrigate, and orientate himself and the planets.

The authors give short accounts of the lives and discoveries of the Egyptian and Greek mathematicians and of the Europeans who, at the time of the Renaissance, read the Latin translations of earlier mathematical discoveries and continued to study and experiment with them.

The results and effect of this accumulation of knowledge is shown at the end of the book in short sections devoted to the measurement of weight, length, money and time.

The book is illustrated by drawings and photographs and contains quizzes and summary illustrations at the end of each chapter.

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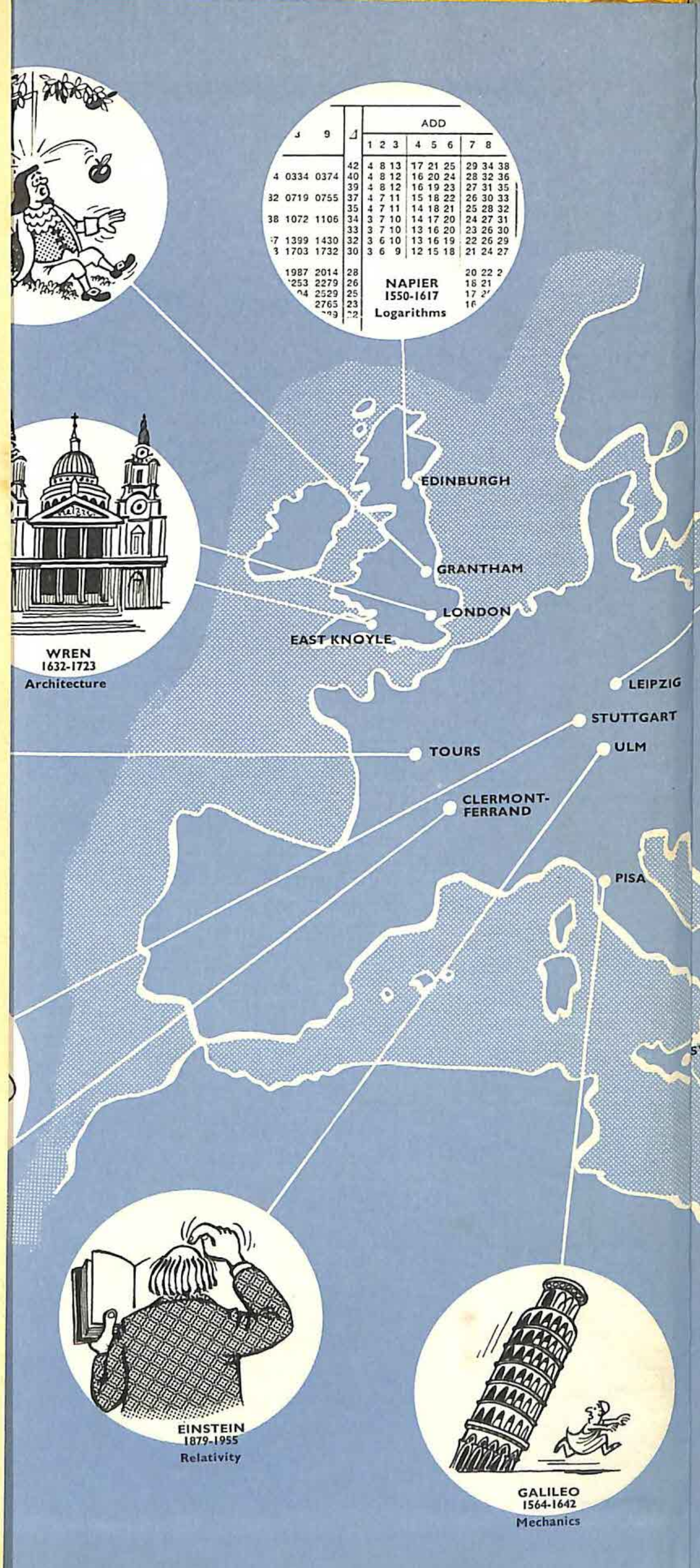

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PLATO
428-348 B.C.
Philosophy
$\frac{dy}{dx} = 6x^2 + 6x$
LEIBNIZ
1646-1716
Calculus
PYTHAGORAS
572-501 B.C.
Geometry
HIPPARCHUS
146-127 B.C.
Astronomy
THALES
640-546 B.C.
Demonstrative Geometry
NICAEA
APOLLONIUS
262-205 B.C.
Conics
SAMOS
ATHENS
YRACUSE
MILETUS
PERGA
ALEXANDRIA (Euclid)
ERATOSTHENES
276-194 B.C.
Geodesy
EUCLID
300 B.C.
Geometry
ARCHIMEDES
287-212 B.C.
Mechanics
SYENE (Eratosthenes)

THE STORY OF MATHEMATICS

STATE INSTITUTE OF EDUCATION
BANIPUR

By
ALAN SHAW
and
KERI FUGE

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Illustrated by
PAUL SELLERS

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PREFACE

Mathematics is progressive and plays a very vital part in our lives.

There are very few discoveries in mathematics which have not been put to good use by man. Its service to mankind is evident in all the things around us, in our buildings, in our work and in our play. We hope that by reading this book you will begin to see mathematics in this light as well as a series of inventions which have developed along with civilisation.

Much research was made by man in order that he might progress; he learnt to count, it became necessary for him to measure his land. His interest in the universe led to the recording of time and the development of the calendar. The invention of the compass enabled him to explore unknown parts of the world, just as today atomic power has opened up even wider fields of exploration.

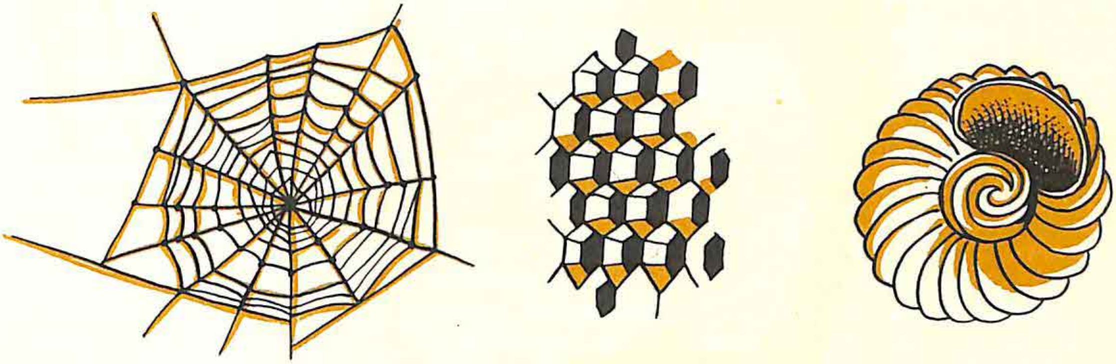
It has not been possible to give more than an outline of the story of mathematics, but we do hope that readers will enjoy what we consider to be the milestones in the history, and that their curiosity will be sufficiently stimulated to want to read more.

The writing of this book was prompted by the evergrowing interest shown by children in the subject. No longer are they content to sit down and work through endless examples. They, like the ancient Greeks, want to know how their mathematics began and what use is made of them. None of this information however would have been available if many gifted writers had not devoted countless hours in research and to the writing of such standard books, as we gratefully acknowledge at the end of this book.

H.A.S.
K.F.
LONDON 1962.

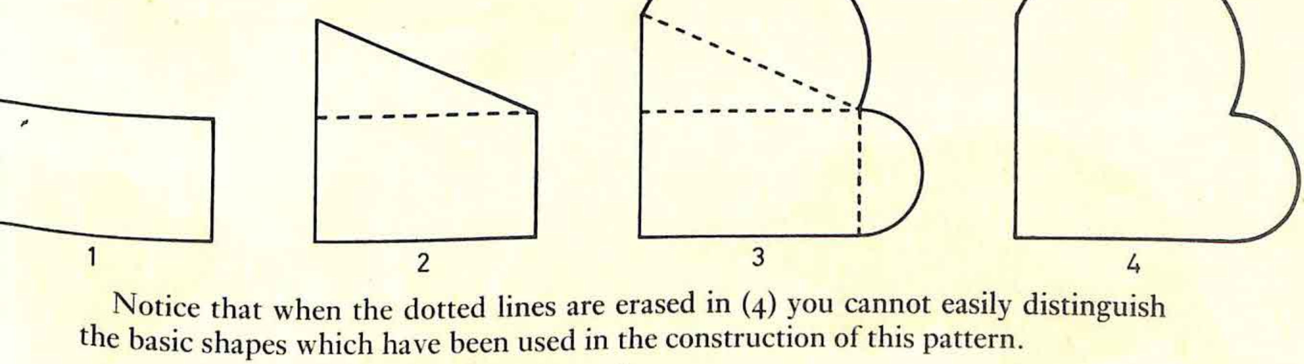
BEFORE MAN WAS BORN NATURE WAS SHAPING THE UNIVERSE

How true is this statement?
The honeycomb; snail's shell; spider's web; are all examples made up of geometric shapes.



You will notice that the honeycomb is made up of a series of cells each having a cross section shaped like a hexagon. The spider's web too is made up of a series of polygons, whilst the snail's shell is in the form of a spiral. This spiral closely follows another of the shapes which we experience in mathematics. The most simple shapes such as the rectangle, triangle and circle you will notice have been used by man in the construction of buildings, in the planning of towns and the construction of furniture. Containers for foods and cleaning materials conform to simple geometric shapes, all illustrating the use that man has made of Geometry.

Some of the shapes which you see around you are made up from a combination of the basic shapes used in mathematics. They are often so complex that their simple bases are not easily recognised, as you will appreciate from the development of these figures starting with the rectangle and combining with it the triangle and circle.



Notice that when the dotted lines are erased in (4) you cannot easily distinguish the basic shapes which have been used in the construction of this pattern.

Look closely at the different types of flowers and you will notice that the petals are arranged in a special pattern, the leaves also forming specific arrangements around the stem. Among other examples, you may examine the shapes made by snow flakes, mineral crystals, the paths described by the planets, the shapes of the spiral nebulæ. Try to extend this list by examining the natural things around you, and see if you can determine what basic shapes are used.

Being exposed to a world so full of pattern and form it seems natural that early man should use some of the simple shapes to decorate his possessions and the walls of his cave, and it is probable that such art led to man's first appreciation of geometry. But of course the first problem of early man would be that of survival.

The hours of daylight he would spend hunting so that he could provide his family with food. During the long dark winter evenings the family would sit around a fire, built to keep them warm and to keep away the wild animals. Tales would be told of experiences during the recent hunt, and the bones from dead animals provided them with the means of making weapons. They had already discovered how to make a needle from the short sharp pieces of bone, and how to use this in making clothes from the skins of the animals they had killed.

THE DEVELOPMENT OF NUMBERS

Two bears were killed today, two sticks were rubbed together to start a fire, his family numbered two.

Three bears would be described as two bears and one bear, and anything bigger than three as much or many. These are all examples of a system of counting used by primitive man. It is probable that counting beyond three developed as man began to keep greater numbers of animals.

You will notice that each number would have a description of two birds, two trees and so on, the object itself being very important. We call these concrete numbers because they refer to specific objects. It is quite possible that you have often used concrete numbers, particularly when you first started mathematics, i.e. three counters added to four counters make seven counters. But there is no doubt that you would soon be writing this problem as $3 + 4 = 7$, using what we call abstract numbers. Many of our numbers becoming abstract because of constant use.

As the idea of thinking in concrete terms gradually disappeared it became necessary to develop a system of numbering. You can imagine that different methods of numbering would develop in all parts of the World often depending upon the environment of the people concerned. It was many hundreds of years after the Stone Age had passed by before a standard method emerged, and this was mainly due to the invention of the printing machine.

In order that a system of numbering can be developed it is necessary to find a base for that system. In our present day system we use 10 as a base, in other words we group objects in tens, or tens of tens which we call hundreds, but early man used 2, while there is much evidence to suggest that 3 and 12 were also very popular as a base.

At the time when the Chinese were using 2 as a base, the people living along the banks of the Euphrates were using 3, and people on the Nile were using 5. The inhabitants by the Nile had realised that use could be made of the five fingers of each hand. Here lies the clue to the system we use today, developing from the mere chance that nature provided us with 10 fingers. The adoption of 10 as a base however, did not occur until hundreds of years after the Egyptians had been using 5 as their base.

There were also systems based on 20 as a group, which probably arose from the counting of fingers and toes, a system which was used widely by the American Indians. One of the largest bases used was 60, a Babylonian system. This system is still used today when we measure time and angles. Other relics of early number

systems which are in use today are 12 inches = 1 ft., 12 pence = 1 shilling. In Wales they speak of one and fifteen over twenty for thirty six; ages in the Bible are given as three score years and ten, both systems using 20 as a base.

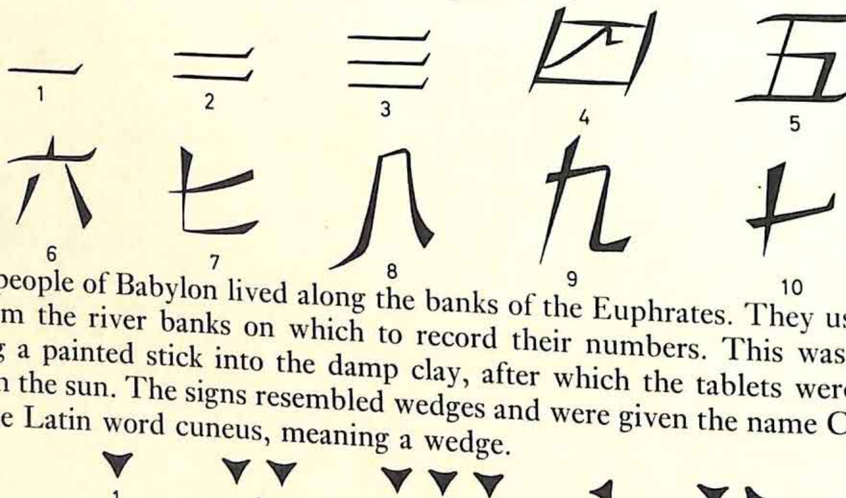
THE WRITING OF NUMBERS

Just as the bases used for number systems varied throughout the World, so the methods of recording numbers varied, the materials used often reflecting the environment of the people.

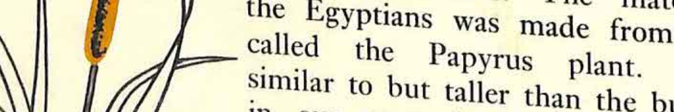
We have seen how early man decorated his weapons and belongings. It would be a natural step for him to record numbers in a similar way by using pictures and lines to represent the result of a hunt.

From the cutting of notches in a stick developed the Talley sticks used commonly in England until the early 19th century. The Talley stick was used extensively by the merchants and traders. The transaction was recorded on the stick, which was then split down the centre, one part being kept by each party to the transaction. To help with the reading of talley marks, each basic group of notches would be made to stand out, probably by a notch cut at an angle across the other marks.

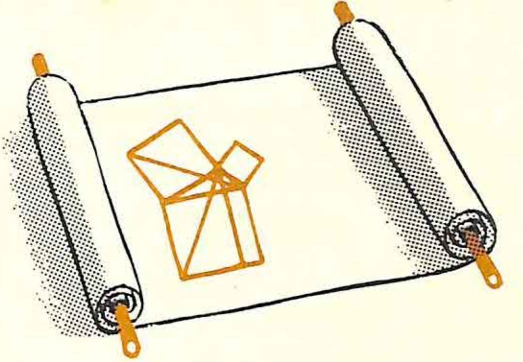
The making of strokes and lines to record numbers is evident in many of the early systems of writing. In China, a wet brush rubbed in a black powder was used to make bold figures on pieces of the palm leaf.



The people of Babylon lived along the banks of the Euphrates. They used damp clay from the river banks on which to record their numbers. This was done by pressing a painted stick into the damp clay, after which the tablets were allowed to dry in the sun. The signs resembled wedges and were given the name Cuneiform from the Latin word cuneus, meaning a wedge.

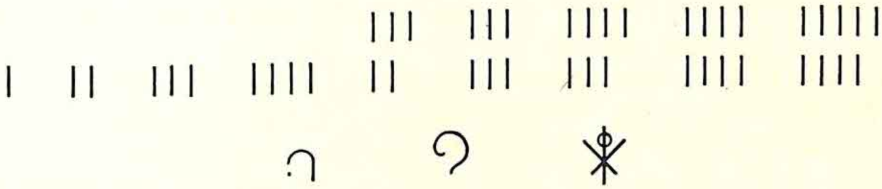


Living in a similar environment along the fertile banks of the Nile were the Egyptians, who had developed another material on which to record their numbers. You will have read in the Bible about Moses in the bullrushes. The material used by the Egyptians was made from such a plant called the Papyrus plant. It was very similar to but taller than the bullrush we find in our swamplands today. This plant was

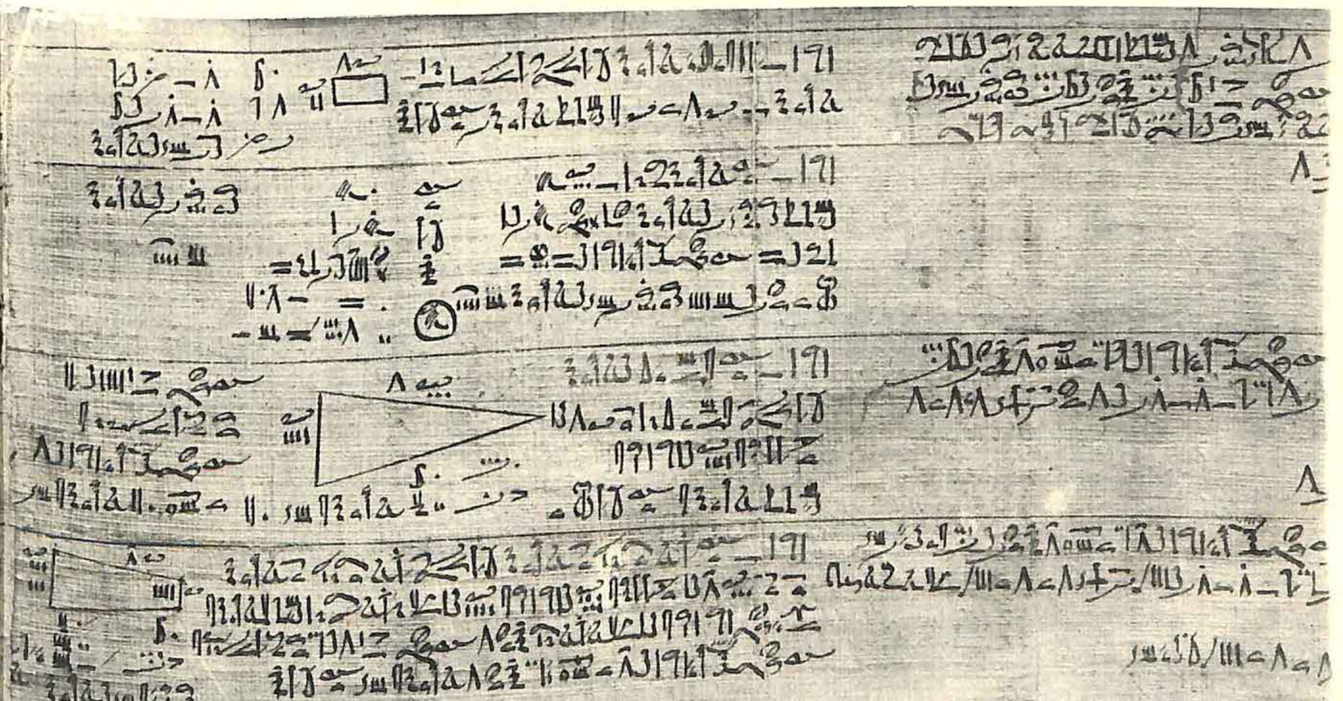


cut into strips which were laid together and pressed, then left to dry. The sap formed a bond between the two layers.

When finished, the papyrus looked very similar to the rough brown paper we use today for wrapping parcels. After it had been dried the papyrus was cut into strips and rolled to form the papyrus scrolls, and on these the Egyptians would write using a black powder mixed with water. It is probable that when Joseph was keeping the records of wheat stored in Pharaoh's barns he used figures such as this.



The oldest book on mathematics in the World was written in approximately 1650 B.C. and remains as one of the most important sources of Egyptian mathematics. The scroll, known as the Rhind Papyrus, is now in the British Museum. It is sometimes referred to as Ahmes Papyrus after the scribe responsible for copying it from the sources existing at that time.

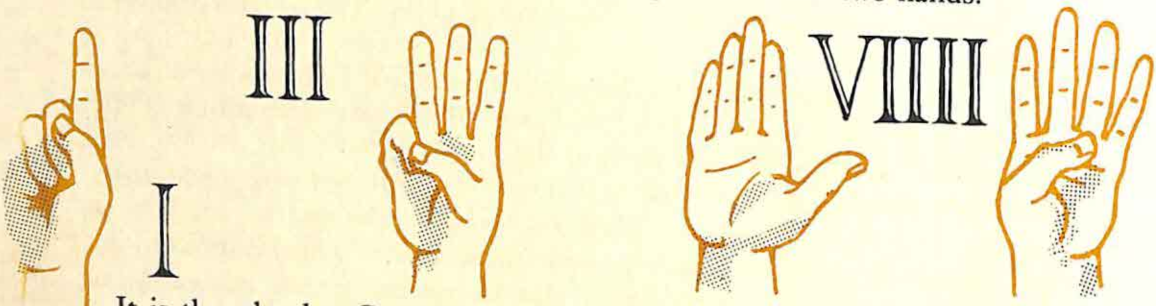


I expect that many of you have seen Roman Numerals used on a clock face or numbering the chapters of a book. Of these early systems we may consider the Roman system is one of the simplest to remember because there has been so little change in it from the simple finger notation.

I II III IV V VI VII VIII IX X

L = 50 C = 100 D = 500 M = 1000

The figures I–IV represented the four fingers and five was represented by V which was a picture of the hand, X being a picture of the two hands.



It is thought that C, which stands for 100, may be made from the first letter of the Latin word centum. The symbol for 1,000 is thought to have come from the Greek letter phi which was written Φ, later developing into M.

By repetition the system was built up and it was not until some time after the Romans that the system was modified to include IV for four and IX for nine. These Roman symbols are found by subtraction, IV is one less than V and IX is one less than X. We use a similar method today in giving the time as 10 minutes to 3 instead of 2.50. The year 1962 would be written as MCMLXII, i.e. M represents 1,000, CM is 1,000 − 100, L is 50, XII is 12.

With this system however it is notable that the numbers were used mainly for recording purposes. If you try to do simple multiplication using these symbols you will find it is far more tedious than with the symbols we use today. Notice also how you miss the Zero, which was not included in the Roman system. The symbol for zero was invented in India and it was certainly used in that country long before it was used in the Far East.

It is from the Hindu characters that our modern figures are said to have originated. As you will read later the Moors invaded Spain, bringing with them a knowledge of Arabic and Hindu numerals which slowly filtered into Europe. Such figures you will recognise as being very similar to those which we use today.

1 2 3 4 5 6 7 8 9 0

The Hindu symbols for 2 and 3 were $=$ and $\equiv$. Try writing these quickly and you will see how our modern 2 and 3 have developed.

For hundreds of years after the Arabs had introduced these new symbols into Europe many people still used the Roman numerals, and it was not until the invention of moveable type that numerals became standardised and the writing of books by hand gradually disappeared.

MECHANICAL AIDS TO COUNTING

Two thousand years ago the Romans wrote their numbers on a wooden tablet, very similar to the slates used by very young children today. The tablet was covered with wax and the numbers marked on it with a pointed stick. After use the wax would once more be smoothed with the stick. The actual calculations however would be made on an abacus, which looks very much like a counting frame. These, of course, varied in different parts of the world.

In some cases the Abacus consisted of a tray of sand, lines being made in the sand, one line to represent units, another to represent tens and so on. Pebbles were then placed on these lines to represent the number. The Latin word for pebble is Calculus and it is from this word that we get our words 'calculate' and 'calculation'.

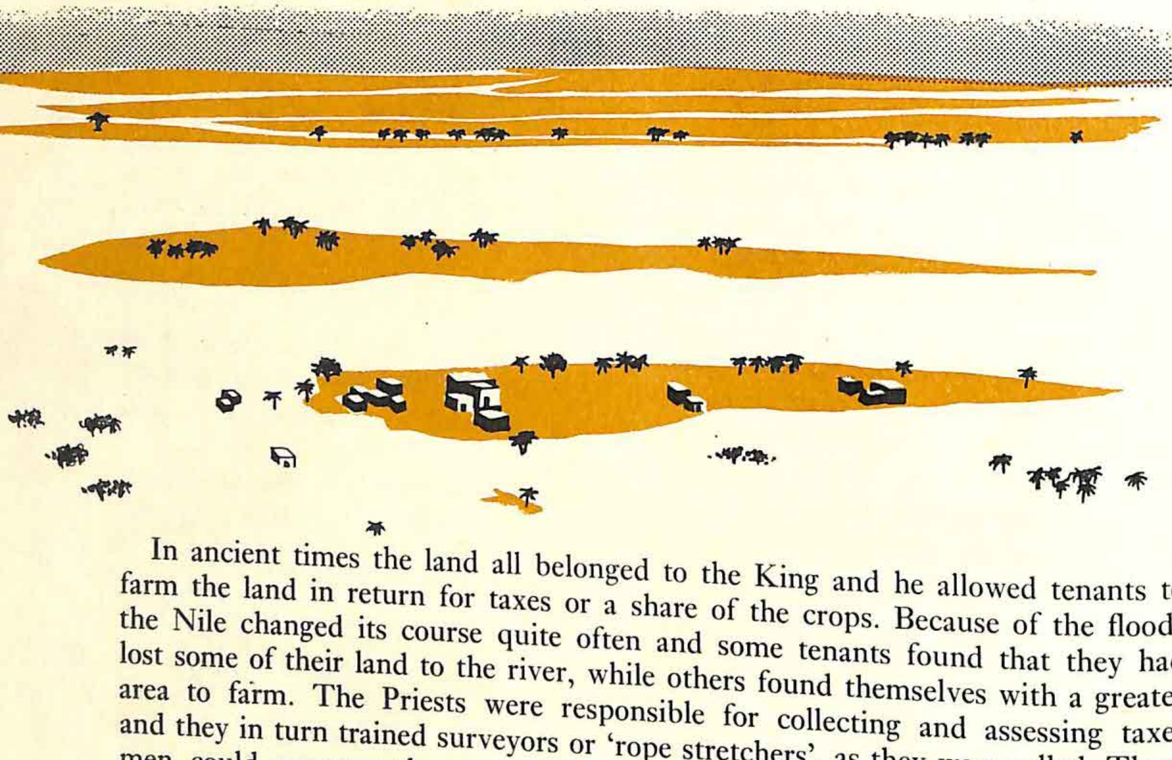
At this time the Chinese used bamboo sticks to represent numbers, and in some parts of the Far East short bone rods were used. About 1,000 years later however they did invent a reckoning board, the suan pan, which is still used in shops and schools today.

The Abacus gradually disappeared from Europe with the introduction of Hindu-Arabic numerals, but again in this modern age we see aids to calculations in the adding machine and computer.

THE WONDERFUL ACHIEVEMENTS OF THE EGYPTIANS

It has been claimed that without the Nile there would be no Egypt. Most of the country is desert and it is divided by the wide flat valley through which the Nile flows to the sea.

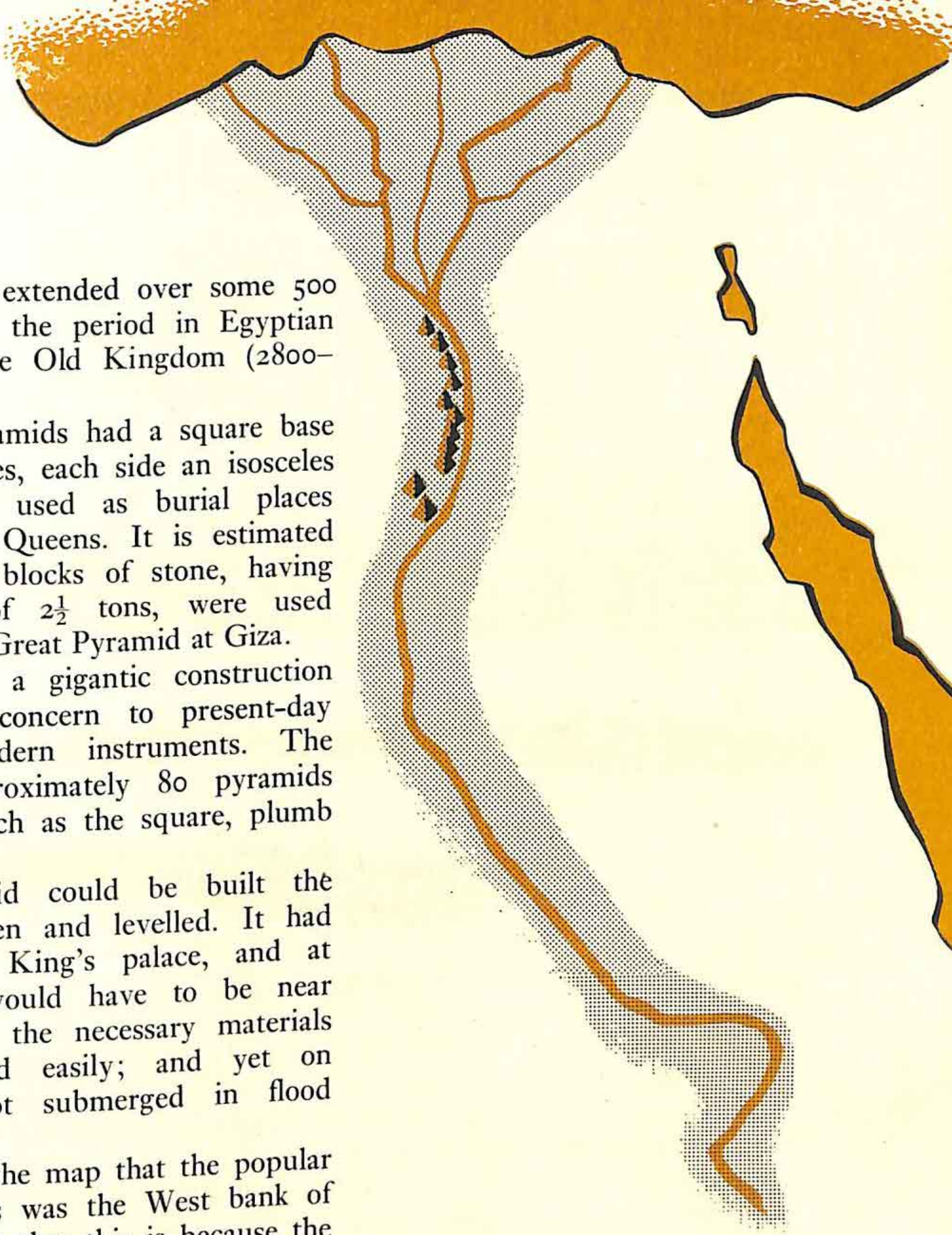
Every summer the heavy rainfall in the hills of Abyssinia swells the Nile, and when the White Nile joins the Blue Nile at Khartoum the river overflows its banks, flooding the valley for hundreds of miles. It deposits a fertile soil in which are grown most of the crops of the country. It naturally follows that the towns and cities of Egypt occupy sites in the Nile Valley which are above the annual flood level. In modern times however a dam has been constructed to provide for the needs of the country throughout the year.



In ancient times the land all belonged to the King and he allowed tenants to farm the land in return for taxes or a share of the crops. Because of the floods the Nile changed its course quite often and some tenants found that they had lost some of their land to the river, while others found themselves with a greater area to farm. The Priests were responsible for collecting and assessing taxes and they in turn trained surveyors or 'rope stretchers', as they were called. These men could measure the area accurately despite their crude instruments. The Priests were also responsible for telling the inhabitants when these floods would occur, and to help them with this task they put posts in the river at intervals so that they could measure the rise and fall of the water level. As always happened however, such devices were kept secret, the posts being well hidden from the common man.

When the land was allocated each year it had to be levelled for cultivation, and being in a desert area where there was insufficient rain to soak the land, irrigation canals had to be made. So you will see the Egyptians gained a lot of experience in the marking out of areas and in the behaviour of water levels, all of which they found assisted them in the gigantic task of building the pyramids.

THE PYRAMIDS



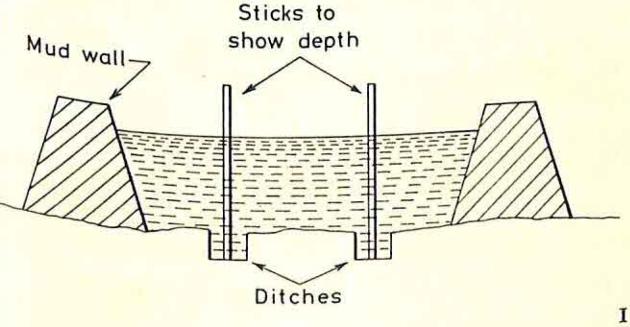
The Pyramid Age extended over some 500 to 600 years, during the period in Egyptian History known as the Old Kingdom (2800–2300 B.C.)

In general the Pyramids had a square base with four sloping sides, each side an isosceles triangle. They were used as burial places for their Kings and Queens. It is estimated that some 2,000,000 blocks of stone, having an average weight of $2\frac{1}{2}$ tons, were used in the building of the Great Pyramid at Giza.

The task of such a gigantic construction would cause some concern to present-day engineers using modern instruments. The Egyptians built approximately 80 pyramids using simple tools such as the square, plumb line, chisel and awl.

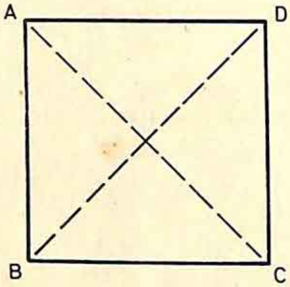
Before the pyramid could be built the site had to be chosen and levelled. It had to be near to the King's palace, and at the same time it would have to be near to the river so that the necessary materials could be transported easily; and yet on land which was not submerged in flood time.

You will see from the map that the popular side for the pyramids was the West bank of the Nile. It is thought that this is because the Egyptians connected death with the setting of the sun.



In order that the site might be levelled it would first be enclosed by a wall of Nile mud. This area was then flooded and ditches dug to a constant depth below the level of the water. Thus obtaining certain datum points. The water was then drained away and the remainder of the site excavated to the level of the ditches.

Mark out a square on a flat piece of ground. How can you check to see if it is a square?

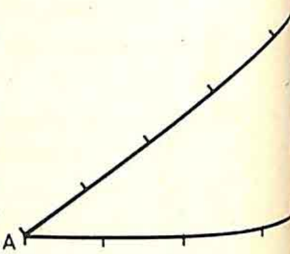


One method would be to make a diagonal check. In this diagram if ABCD is truly square then AC should be equal to BD. It was not always possible to find suitable flat sites for the pyramids; often, too, there would be a mound or large rock protruding from the centre. The Egyptians overcame this by using the 'rope stretchers' method of constructing a right angle. This method is based on the fact that a right angle is formed when the sides of a triangle are in the ratio of 3 : 4 : 5.

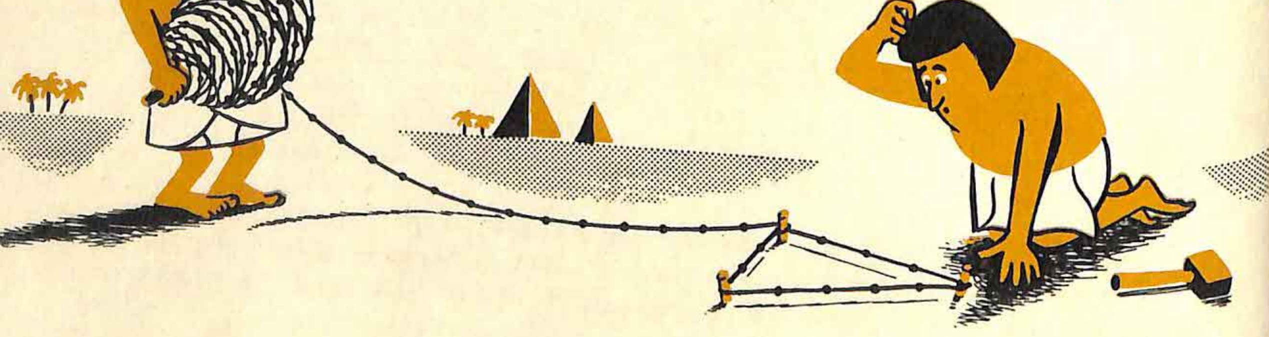
Tie some knots at regular intervals in a length of rope. Peg out AB equal to four units, then lay BC—three units, roughly at right angles to AB. Swing round the remainder of the rope until AC is exactly five units. Pull the sides of the triangle taut. Angle ABC is now a right angle.

This method of marking out right angles was also used by the Babylonians, another race who lived under similar conditions to the Egyptians.

When we realise the difficulties of marking out the sites for the pyramids and the methods used, it is remarkable that the errors were so small. The length of each side of the Great Pyramid at Giza is 756 ft., its base covering an area of $12\frac{1}{2}$ acres. It has been discovered by archeologists that the average error on each side is less than 8 inches. That is 8 inches in 9,072 inches. It has also been found that the ratio of the height to the perimeter is the same as the ratio between the radius and the circumference of a circle.

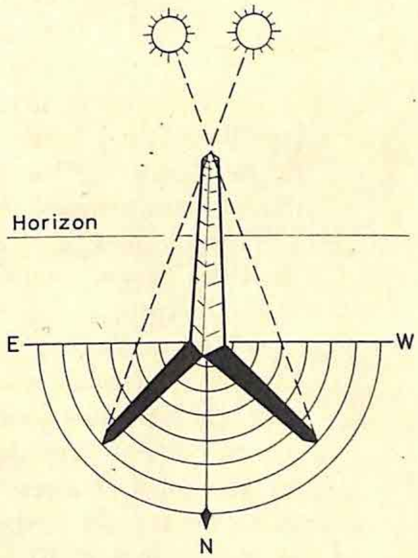


Can you say at which corner of the triangle the right angle should occur in the illustration below?



Look at a map of the Near East and you will see that Egypt is situated between latitudes 22° and 32° N. In such an area the Egyptians would enjoy clear starry nights and brilliant sunlit days, so it is understandable that they became curious about the mysterious universe. The Egyptians, following their various trades, depended on the priests and wise men for recording time and keeping the calendar. When any major construction or irrigation project was formulated, the priests showed the natives how to do it and were responsible for directing the labour.

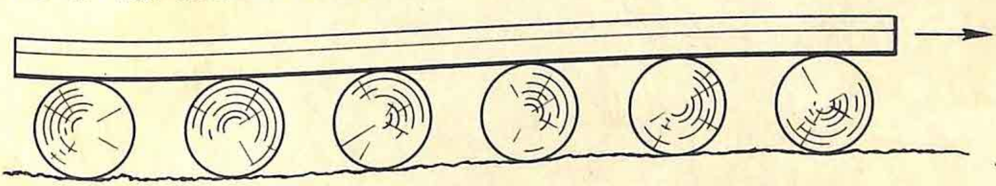
For many years the priests had recorded observations of the movements of the sun, noting where it rose and set each day. To help with these observations they erected stone pillars called obelisks. At the foot of the obelisk concentric circles were drawn to assist with measuring the lengths of shadows cast by the sun.



They discovered that by noting the position of the shadows, before and after noon, they could bisect the angle formed. This gave the position of North, after which it was a simple matter to find the other cardinal points.

This was one method used for orientating the pyramids which were built so that each of the four sides faced North, South, East or West. So accurate were these observations that the average error of the faces of the Great Pyramid and the true directions of North, South, East and West was less than one degree.

The average weight of stone blocks used in the construction of the Great Pyramid was $2\frac{1}{2}$ tons, one of the biggest weighing as much as 200 tons. These huge blocks of stone were transported from the quarries to the site by means of barges, the process usually being carried out during the period of the flood. The two main stones used in these constructions were limestone for the outer casing, and granite for the core. Limestone, the softer rock, was quarried with chisels and wedges from the area around Tura. Because of its strength granite was used to form lintels over corridors and the burial chambers. It was obtained from the Aswan quarries.



To move the stone over land a sledge was used and pulled along on rollers, a liquid being thrown on the ground to lessen friction. You can simulate this procedure by placing your ruler on a number of pencils and rolling it along your desk.

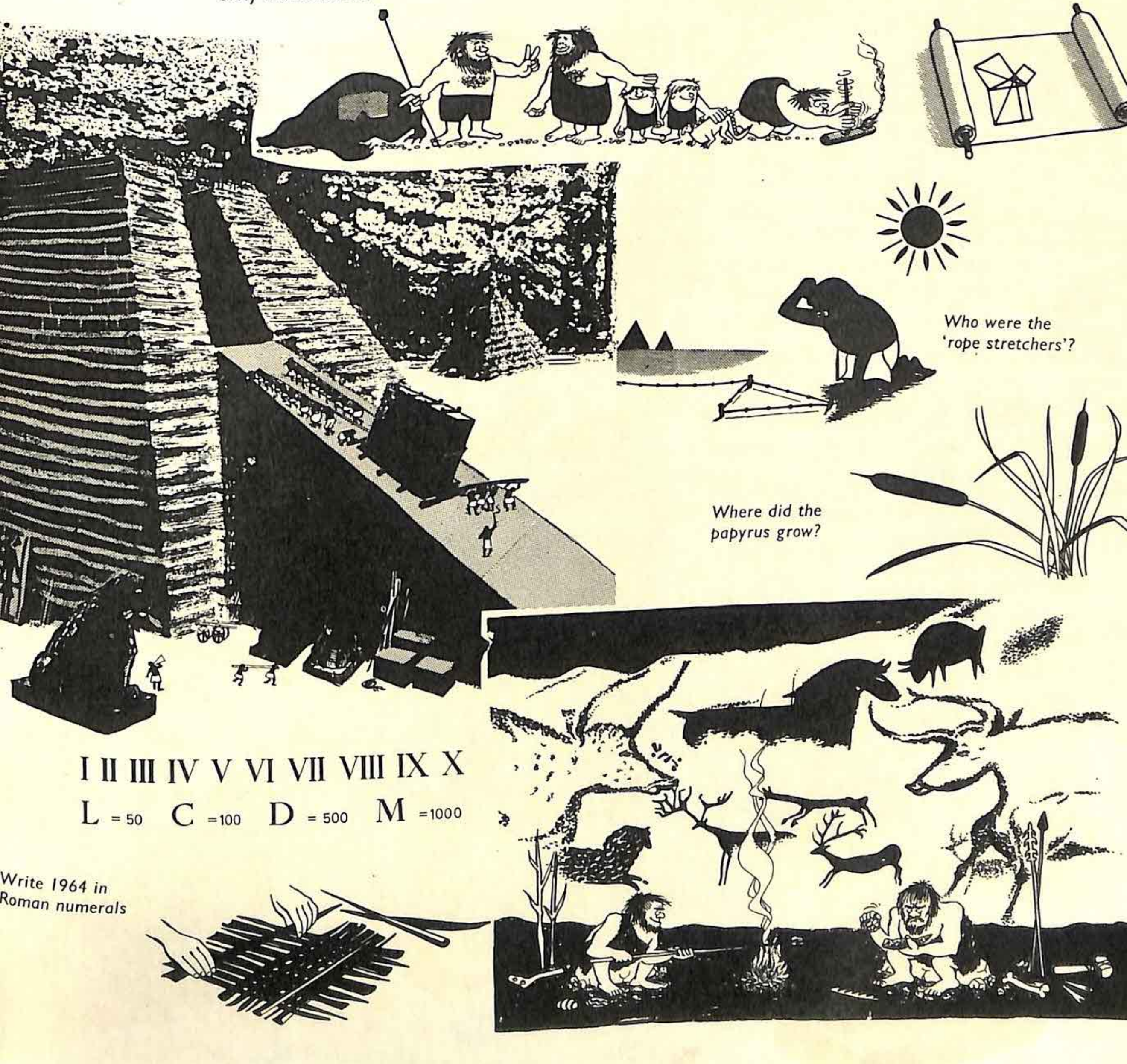
The actual method of construction of the pyramids is still uncertain but we think that a ramp was used for raising the stone from one level to the next. The blocks of stone were dressed down and placed in position, forming a square base. Rubble and earth from the locality would be used to make a ramp up to the level, then another layer of stone would be hauled block by block up the ramp and placed squarely in position. This process would be continued, the ramp growing higher as each layer and casing was completed, until the apex was reached. Ropes of palm fibre, levers and rollers as well as unlimited labour would be used extensively on this task.

It has been estimated that the sides of the Great Pyramid were inclined at an angle of 52°, in which case it would have been 480 feet high.

With the pyramid completed, the ramp, which often extended over the four faces, was dismantled. The sides of the pyramids were finally dressed and smoothed as this work progressed.

The methods mentioned in this chapter had been used by the Egyptians for many years. The priests, who directed operations, did not teach the craftsman the principles of the methods used, they merely instructed them in their use. All theoretical knowledge known by the priests and scribes was kept secret.

ou explain the relationship between these illustrations and the development of early mathematics?



THE EARLY MATHEMATICIANS

It has often been said that the Greeks were the first mathematicians, but prior to the period we are about to consider the Egyptians were already familiar with the properties of the rectangle, triangle, circle and pyramid, as well as with a system of decimals and fractions. They had made practical use of the movement of heavenly bodies for finding direction and water had been used for finding levels. Squares could be set out with great accuracy and areas of land measured.

THALES (624–548)

Because of his important discoveries Thales was later honoured as being one of the 'Seven Sages of Greece', and he has also been described as the first man worthy of the title 'Man of Science'.

As a young man he was a merchant who travelled widely in the Middle East, collecting any scraps of knowledge that came his way. He is reputed to have made a fortune when still young, retiring to spend the rest of his life studying the stars and laying the foundations of mathematics. There are several tales of how Thales solved problems which confronted him. One is said to have happened when he was travelling with a pack of mules. One of the mules fell into a stream and was most elated when the load of salt which it was carrying dissolved.

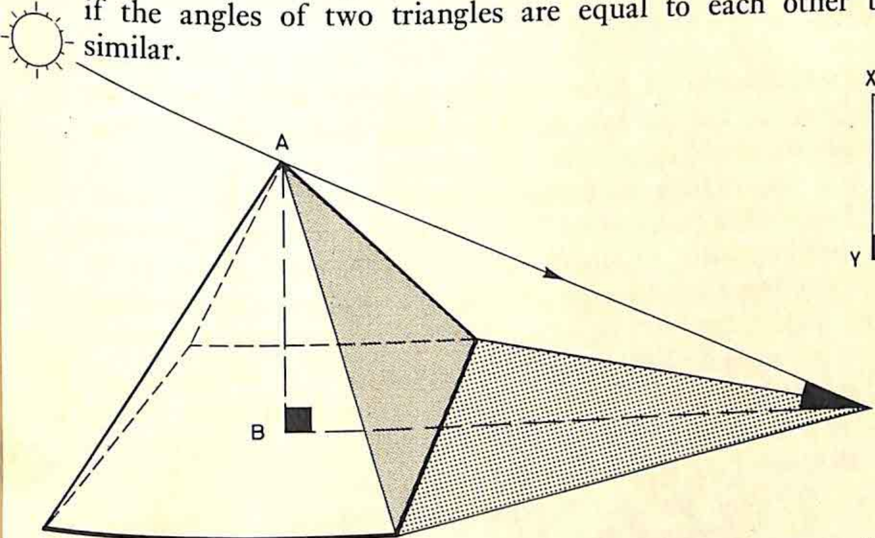
After that it proceeded to roll in every stream or pool it would find, causing Thales some concern. Thales soon solved the problem by making the next journey with a load of sponges.

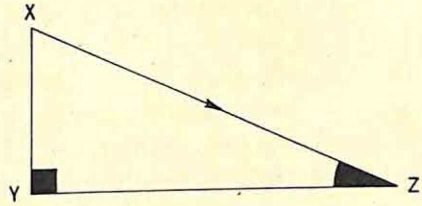
While in Babylon he became fascinated with the stars and transformed what had been little more than a list of observations into the beginnings of Astronomy, one of his most spectacular forecasts being the eclipse of the sun in 585 B.C. The Greeks were at war at this time but they were so impressed that the battle

was abandoned and peace followed for many years.

Despite his success as an astronomer, Thales' chief claim to fame was as a Geometer. At this time people were only interested in doing things practically as has been shown in the chapter on the Egyptian builders. Thales refused to accept every day occurrences and set out to find a reason for everything that happened. He was not satisfied if something worked unless he could prove why it worked.

It was during his journeys in Egypt that Thales used the theory of similarity for finding the height of the Great Pyramid. One law of similarity states, that if the angles of two triangles are equal to each other then the triangles are similar.





Comparing triangle ABC and XYZ
$\angle$ ABC = $\angle$ XYZ (rt. angles)
$\angle$ ACB = $\angle$ XZY (altitude of sun)
$\angle$ BAC = $\angle$ YXZ (third angle of Δ)
The triangles having equal angles are similar ΔABC III ΔXYZ

$$\frac{\text{Height of Pyramid}}{\text{Length of Shadow}} = \frac{\text{Height of Stick}}{\text{Length of Shadow}}$$

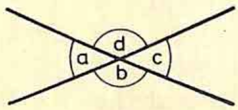
$$\frac{h}{200} = \frac{12}{5}$$

$$5h = 2400$$

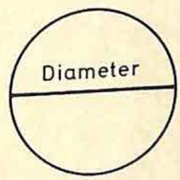
$$h = 480 \text{ ft.}$$

OTHER DISCOVERIES OF THALES

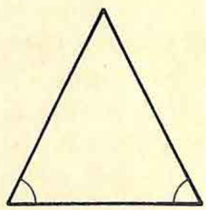
1. $\angle a = \angle c$, $\angle b = \angle d$



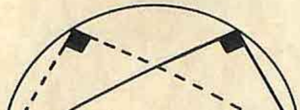
2. Any diameter bisects the circle.



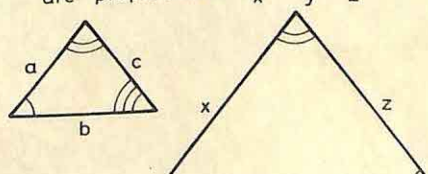
3. Base angles of Isosceles triangle are equal.



4. Angles in semi-circle are right angles.



5. Sides of similar triangles are proportional $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$

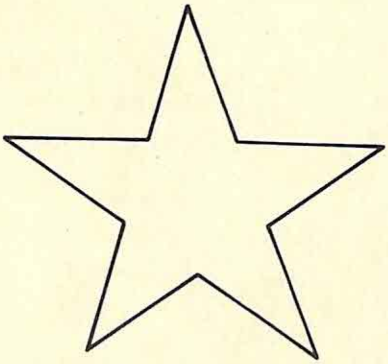


PYTHAGORAS (572–507 B.C.)

Pythagoras was born on the island of Samos, an island about the size of the Isle of Wight and situated off the coast of Turkey. As a youth he studied under Thales who advised him to visit Egypt.

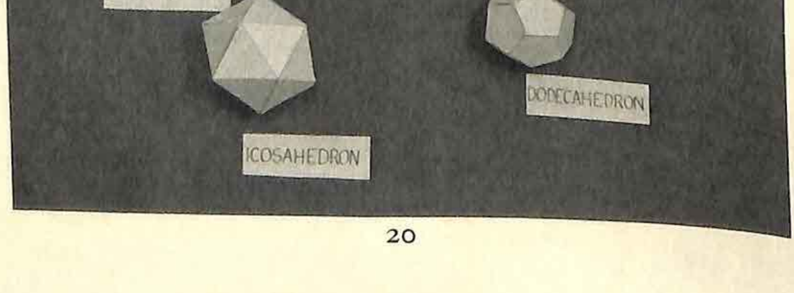
In 529 B.C., Pythagoras arrived at the house of his friend Milo in Crotona, where he soon earned the reputation of being a learned man and an outstanding orator. The wealthier Greeks flocked to hear him talk of his travels, and even the women risked their lives to come and listen to him.

Eventually his followers formed themselves into a secret society known as the 'Order of Pythagoreans', choosing for a badge the five-pointed star. They made an oath not to reveal the secrets of the brotherhood, sharing all their belongings and ideas with one another, but attributing any new discoveries to their leader. It is said that Hippasos, one of the brotherhood, revealed the method of splitting the sphere into 12 pentagons and soon after he was mysteriously drowned at sea.



The brotherhood loved to give mystic meanings to mathematics. Odd numbers were described as being divine and known as male, whereas the even numbers were described as being earthly and representing the female. One represented reason, four—justice, five, being a combination of the first even and the first odd number—marriage.

The basic shapes of the equilateral triangle and square led to the discovery of the five regular solids:—the cube, the Tetrahedron—a triangular pyramid having four equal faces, the Octahedron—which has eight faces taking the form of equilateral triangles, the Icosahedron—having twenty equal faces and the Dodecahedron—which has twelve equal faces each one a pentagon.



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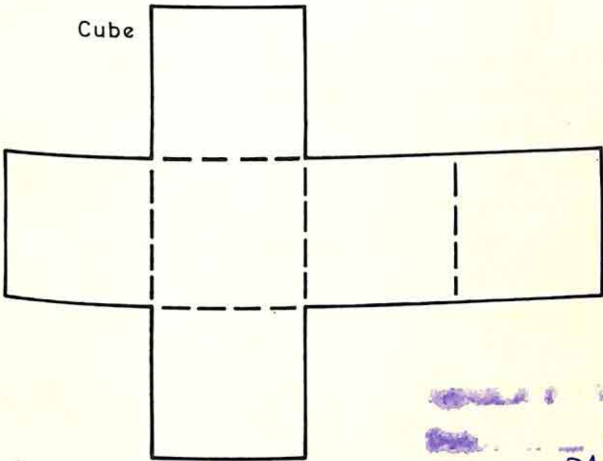
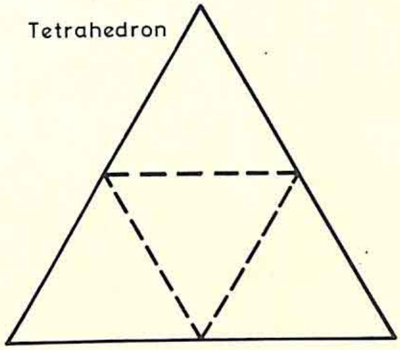
To the Pythagoreans these solids represented the elements of the universe. Tetrahedron—Fire, Cube—Earth, Octahedron—Air, Icosahedron—Water and the Dodecahedron—the Heavenly Sphere.

Make copies of these plans of the solids out of stiff card, fold along the dotted lines and fix together with sticky tape.

Cube

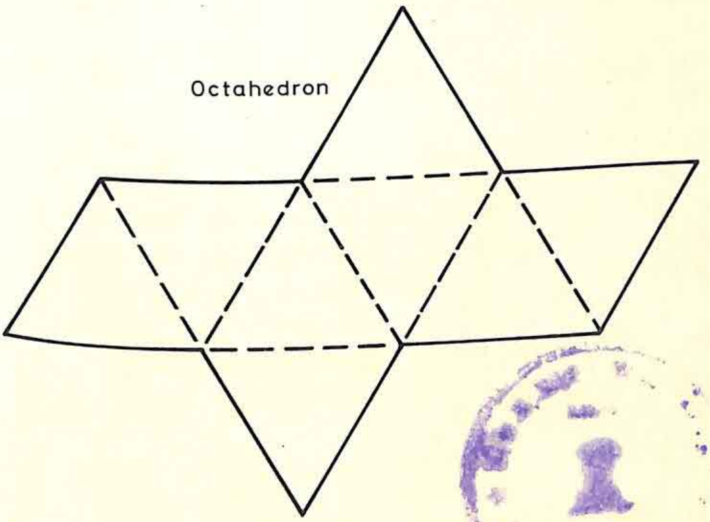


Tetrahedron

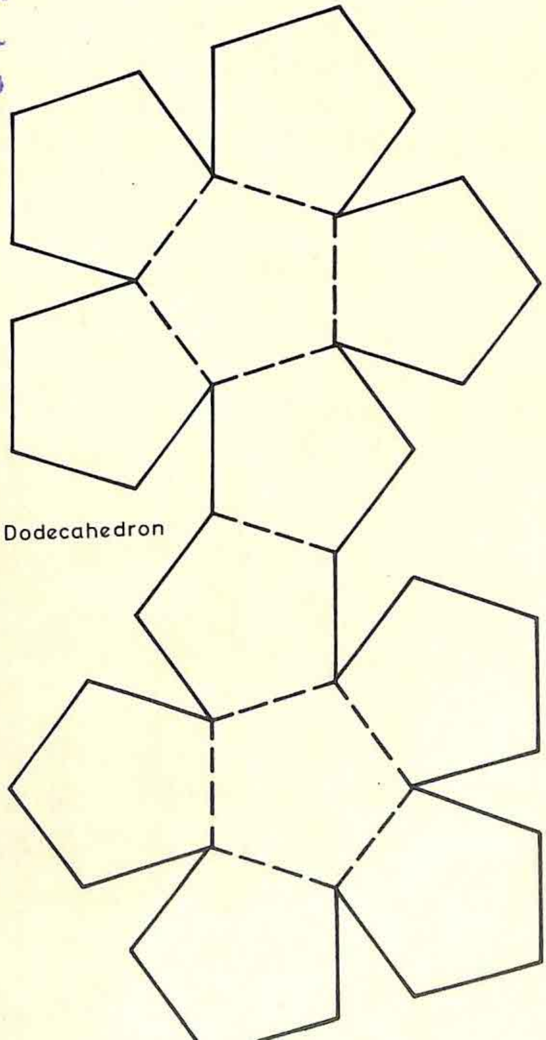


27·8·93
7342

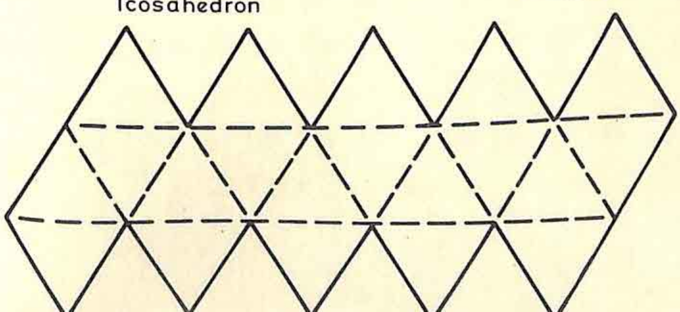
Octahedron



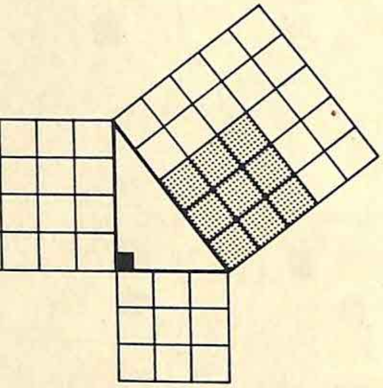
Dodecahedron



Icosahedron

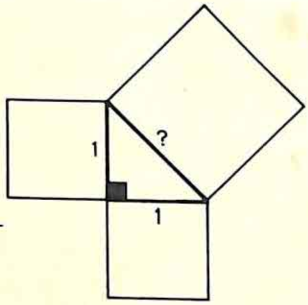


The shadows cast on the tiled floors of Grecian buildings may have led to the famous theorem of Pythagoras, which tells us that a square drawn on the longest side of a right angled triangle (the hypotenuse) is equal in area to the sum of squares drawn on the other two sides.



This theorem explains why the rope stretchers were able to construct their right angles with the 3, 4, 5 triangle.

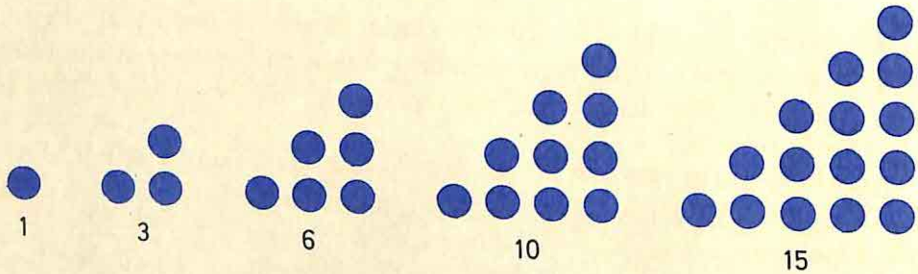
The Pythagoreans were very interested in this discovery but were puzzled when they applied the theorem on the triangle shown below.



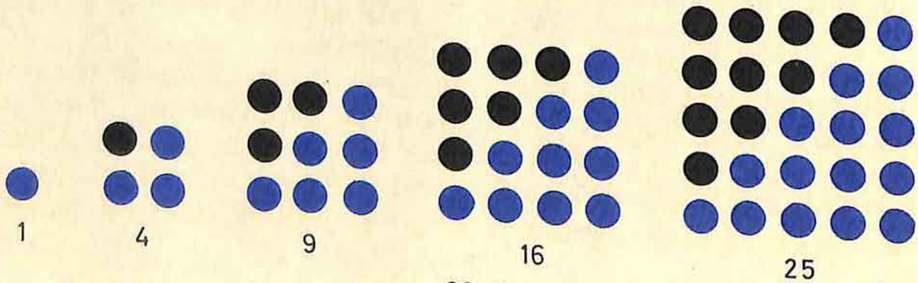
They had no means of finding the value of $\sqrt{2}$ or $\sqrt{3}$ or of any other number which failed to work out exactly.

The Pythagorean geometry was very closely related with their arithmetic. This interest in numbers and geometric patterns led them to build up patterns from numbers.

Use a number of counters to form triangular shapes as shown below.

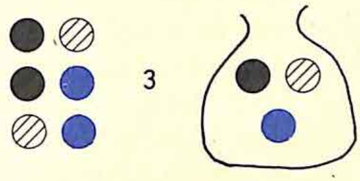


The Pythagoreans found that by combining successive triangles they formed squares.

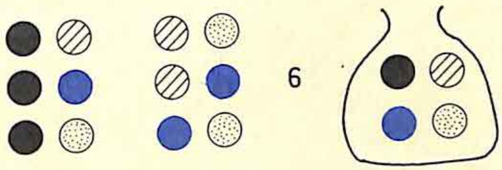


The illustration shows three different coloured counters in a bag.

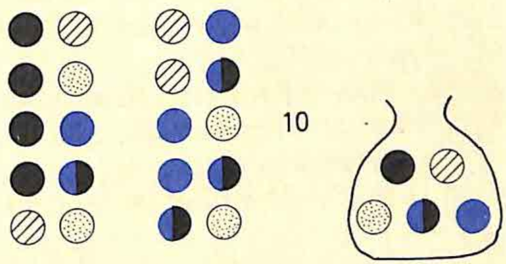
There are three ways in which we can select any two different counters from the bag.



There are six ways in which we can select any two different counters from this bag.



There are ten different ways in which we can select any two different counters from this bag.



If you continue this practice with six counters you will see that the number of selections is 15. Notice that these numbers follow the pattern of the triangular numbers, and may well have been the starting point for theories on number series and probability.

We have seen the Pythagorean interest in geometry and number, but their interest also included music. The harmonic progression and pitch of strings of different lengths were known to the brotherhood.

Studying the shadow cast by the earth on the moon, they realised that the earth was a sphere, a fact that was not generally believed until the 15th and 16th century.

At the time that Pythagoras was killed the brotherhood was very powerful and feared by many of the people, but by continual persecution it was finally forced to disperse, leaving behind a wealth of mathematical discoveries.

About a century after the death of Pythagoras, Greece contained two powerful races who were struggling for power, just as we have two major forces in the world today. Sparta was a slave state where feats of strength and fighting were regarded as being of prime importance, whilst in Athens, which was a democracy, the stress was placed on learning, and it was here that a great philosopher was born.

PLATO (430–349 B.C.)

Like most of the great scholars, Plato spent a number of years travelling in Egypt and Asia Minor and one day he found a manuscript left by Pythagoras in Italy. When Plato retired he established an Academy in Athens where good use was made of his experiences and knowledge.

Above the entrance to the Academy was the inscription 'LET NO MAN IGNORANT OF GEOMETRY ENTER MY DOOR' which sums up Plato's belief that by studying geometry the mind could be trained to tackle problems in an orderly and direct way. This he claimed was essential for all men intending to become leaders. One person who may have heard of this theory was a small-town lawyer aged about 40 who eventually mastered the first six books of Euclid's Elements. He was Abraham Lincoln, the famous president of the United States of America.

Plato spent a great deal of his time seeking a way of life which would do away with war. In the field of mathematics he had little use for practical methods unless there was a way of proving why the method worked. He believed that mathematics should be developed for its own sake, and if it was of practical use then that was of secondary importance, although he did say that when teaching mathematics both amusement and pleasure should be combined to make it interesting. When attempting to prove any problem by drawing Plato insisted on the use of the compass and ruler only. His attempts, and the attempts of other mathematicians in solving the classic problems of trisecting an angle, constructing a square equal in area to a given circle, and drawing a cube twice the size of given cube, led to many mathematical truths being discovered.

Plato fell into the same trap as Thales and Pythagoras by giving mystical meaning to geometric shapes, i.e. equilateral triangle—earth, right angled triangle—water, scalene triangle—air, isosceles triangle—fire.

In the previous pages we have seen how geometry began in Egypt, and was carried to the Ionian Isles, spreading into Italy and then to Athens. In Greece it became the most important cult of the time. The scene then changed back to Egypt centring around Alexandria, the famous city and port created by Alexander the Great.

When Egypt was conquered by Ptolemy Soter he made Alexandria the capital and founded the Great Library, from which seat of learning another mathematician emerged.

EUCLID (ABOUT 365 B.C.)

All that is known about Euclid has been put together from references made to him by various writers. It is suspected that he was a Greek and that he was invited to Alexandria to open up a mathematical school. Lecturing at the Library he had access to the greatest collection of books in the world.

Euclid's Elements consisted of thirteen scrolls. He collected together all the confused pieces of geometry known at this time, arranging the material in order, adding problems on the theorems and filling in any gaps himself. The first six of these scrolls included the properties of triangles, parallelograms, circles, polygons and the proportional division of lines.

The Elements were translated into Arabic about 800 A.D. and brought to Europe in 1120 when Altred of Bath made a Latin translation. In 1570 when Shakespeare was about six years old the Elements were translated into English, the contents of the first six scrolls being used in Geometry books up to 50 years ago.

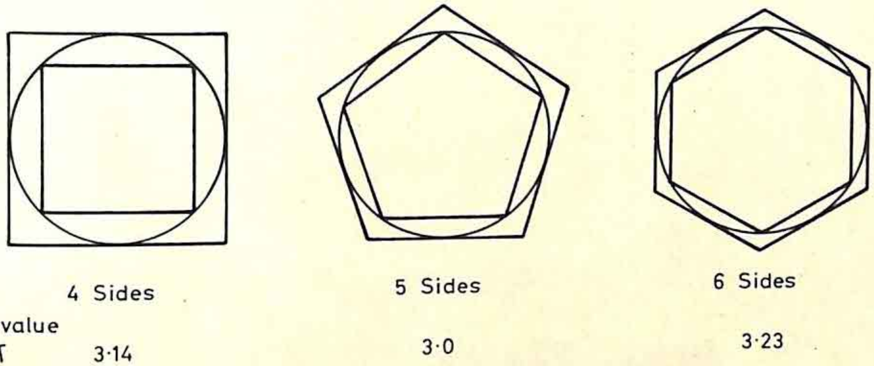
ARCHIMEDES (287–212 B.C.)

Archimedes has often been described as the greatest mathematician of the ancient world. He spent part of his life at the Great Library of Alexandria but completed his work in Syracuse, the capital of Sicily. Unlike other Greek mathematicians he made practical use of mathematics although he often considered it beneath his dignity to do so.

The King commissioned the court goldsmith to fashion him a crown from a block of gold, but when the crown was produced the King suspected that some of the gold had been replaced by a cheaper metal. Archimedes was charged with proving this, the solution coming to him when he was sitting in his bath. He was so excited that he ran down the street shouting Eureka! Eureka! meaning, I have found it. The method he used for solving the problem is based on his famous principle.

'If a body is immersed in water the loss in weight is equal to the weight of the water displaced'.

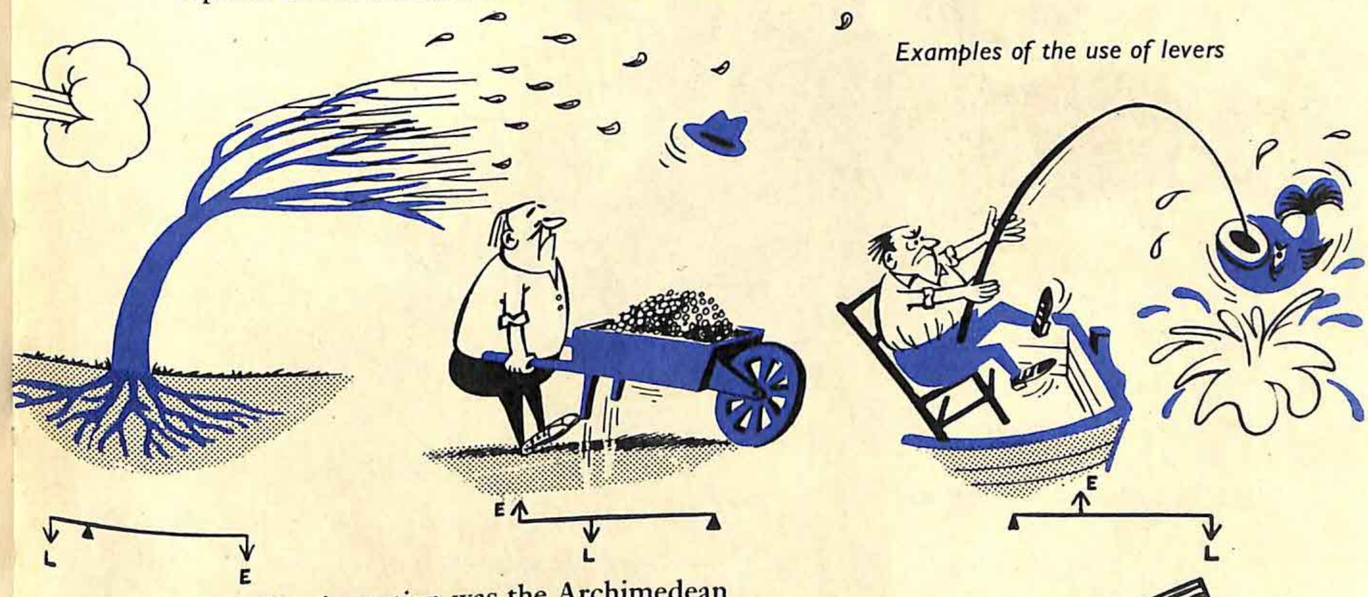
In 240 A.D. Archimedes calculated a value for π (pi) by drawing various polygons inside and outside a given circle.



He eventually arrived at a value of π which was greater than 3.14 but less than 3·142 which was quite an advance from the value of 3 used by the Hebrews and Babylonians.

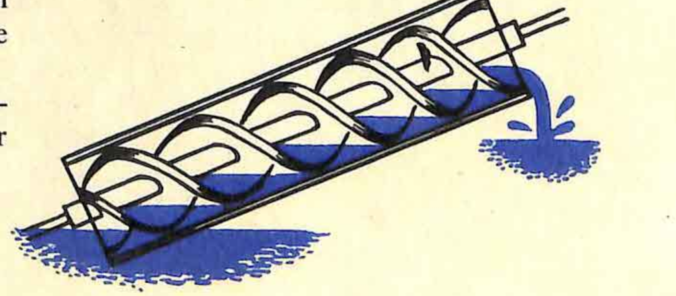
Archimedes found ways of moving great weights by a system of wheels, pulleys and levers. The system of levers formed the basis of statics for many years to come. He is reputed to have said that by such means he could move any weight. 'Given a place on which to stand and a fulcrum I could move the earth'. A fulcrum is a point about which a lever can be turned.

Examples of the use of levers



Another invention was the Archimedean screw used for raising water from one level to another.

This consisted of a helical screw, encased in a cylinder, which raised the water when it was turned.



Battle scene with Archimedes' inventions.

His ingenious inventions were used by the Greeks in war.

The Romans feared these inventions so much that they fled at the least sign of any war machine, but they did eventually take the city. On this day Archimedes was meditating over a problem which he had drawn in the sand. He was so engrossed in this problem that when approached by a Roman Soldier, he said 'Go away. Don't spoil my circles.' The soldier, thinking these to be words of insult, rushed at Archimedes and killed him. So ended the life of a genius.

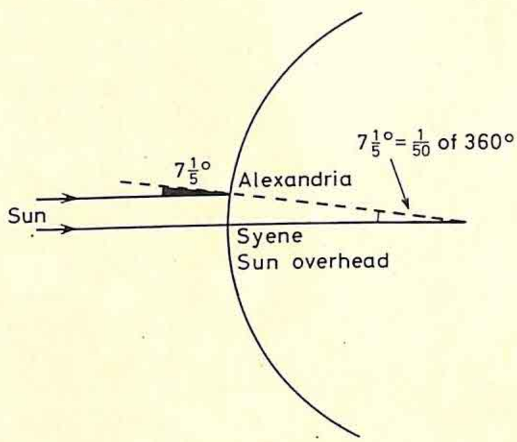
Among other mathematicians attracted to the Library was a friend of Archimedes.

ERATOSTHENES (274–194 B.C.)

Eratosthenes was born at Syene and educated in Athens. He later lectured at the Library, becoming nicknamed 'Beta', the second letter of the Greek alphabet. Some say this was because he was considered a second-rate lecturer; another theory is that his room was labelled 'Beta'. His chief contribution was the calculation of the earth's circumference. This he did at noon on the 22nd June—the summer solstice—when the sun was overhead at Syene, which was on the Tropic of Capricorn.

From records and observations he knew there was no shadow at this time in Syene; but when measuring the shadow at Alexandria he found that the sun's rays differed by $7\frac{1}{5}°$ from the vertical.

Knowing that Syene was 480 stadia south of Alexandria he was able to calculate the circumference of the earth.



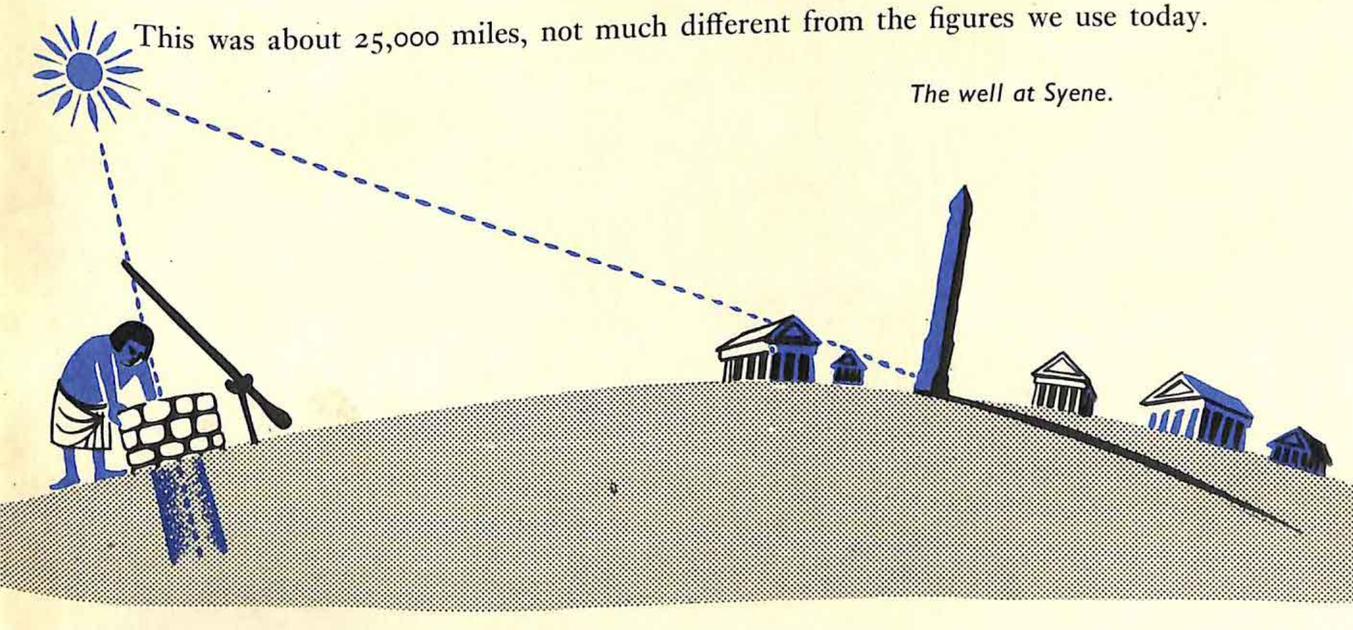
$7\frac{1}{5}° = \frac{1}{50}$ of 360°

$7\frac{1}{5}° = \frac{1}{50}$ of 360°

480 stadia $= \frac{1}{50}$ of the earth's circumference

$\therefore$ Circumference of the earth $= 50 \times 480$ stadia $= 24{,}000$ stadia

This was about 25,000 miles, not much different from the figures we use today.



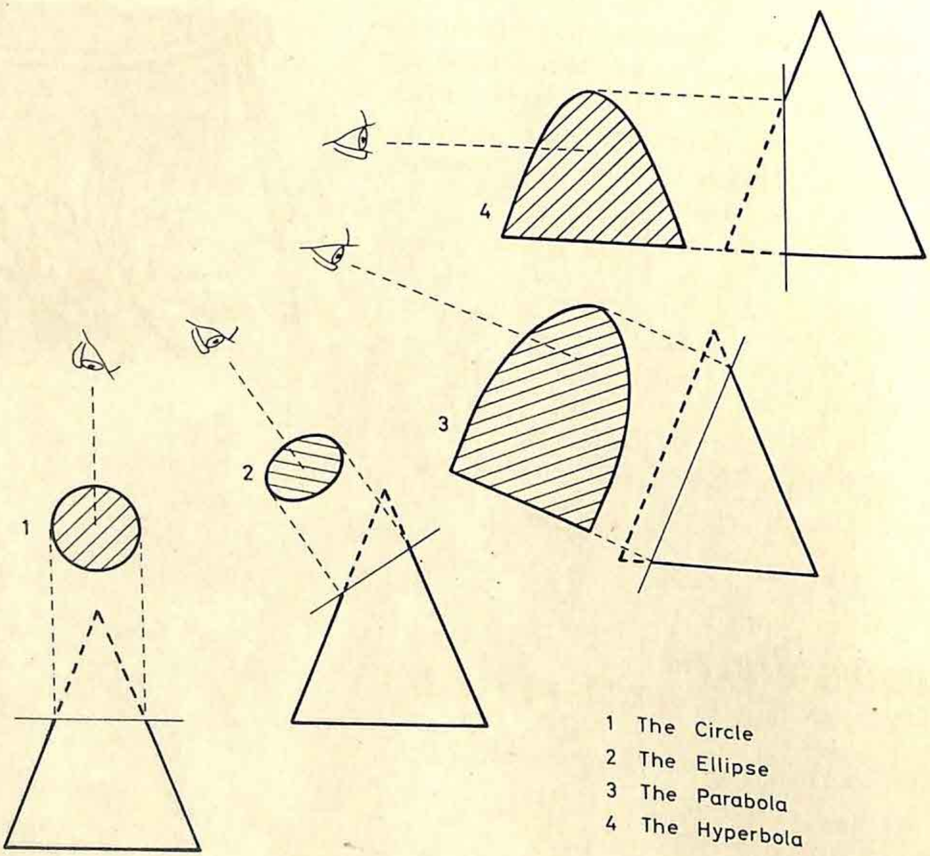
The well at Syene.

APOLLONIUS OF PERGA (225 B.C.)

We do not know a great deal about the life of Apollonius except that he was a student at Alexandria under the successors of Euclid, whose pattern of life he followed.

Apollonius made records of mathematical knowledge and he spent much of his time reasoning out his theories on conics. For this work he is considered to have contributed more to mathematics than Euclid.

When a cone was cut at various angles the shapes of the planes conformed to certain shapes, the ellipse, the parabola and the hyperbola.

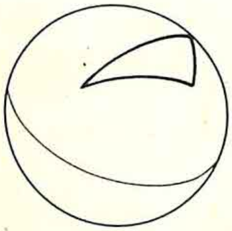


The conic sections obtained by cutting a cone at various angles.

With the death of Apollonius came the end of a great mathematical era which was not to be revived until the 17th century, except for some of the mathematicians illustrated on the opposite page.

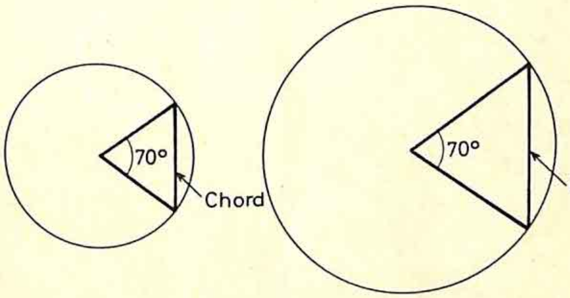
HIPPARCHUS (180–125 B.C.)

The first astronomer to make a map of the stars.



Spherical Triangle used for mapping the stars.

He made a table showing the relationship between the length of a chord and its angle at the centre of a circle. From these tables he was able to calculate the length of chords in bigger or smaller circles.

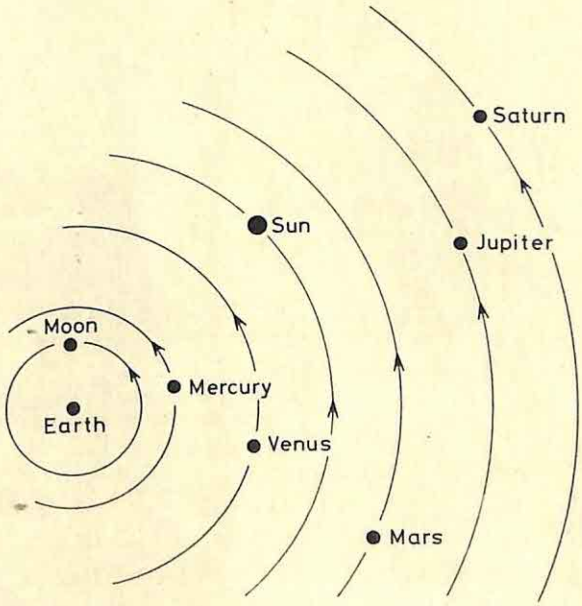


HERO (62 A.D.)

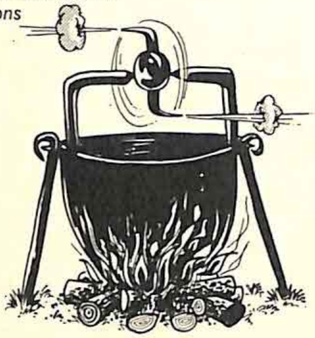
Applied mathematics to his inventions, i.e. the steam engine.

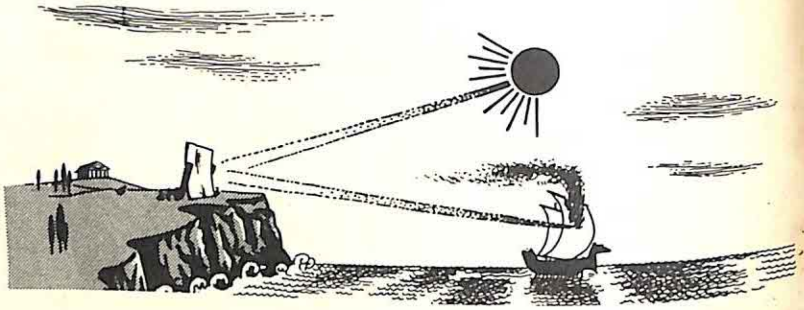
CLAUDIUS PTOLEMY (100–168 A.D.)

Thought everything revolved around the earth.

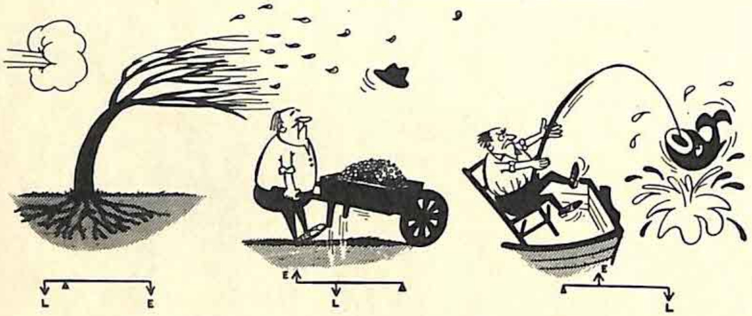


With which early mathematician were these illustrations connected?

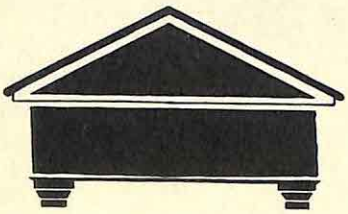


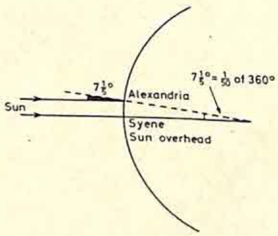


Find out some other examples of the use of the lever.

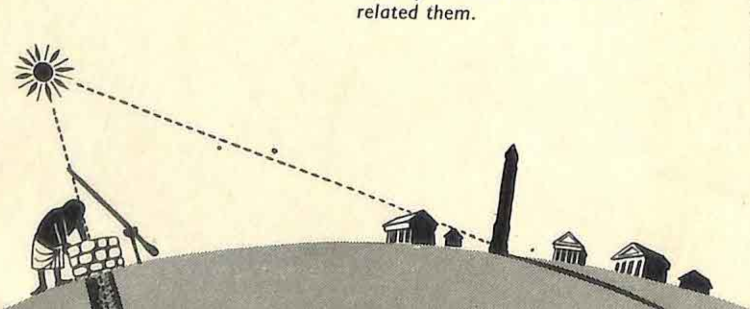


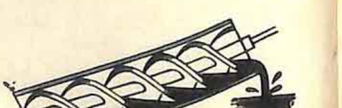
For which geometrical proof is Thales remembered?





Name the different types of triangle and say to which elements Plato related them.





So far we have traced the growth of Mathematics through the period when Greece was the most powerful country in the world, their way of life and their beliefs encouraging learning and the discovery of new facts. With the rise of the Roman Empire, Law and Government were given far more significance. The Greek era finally came to an end in the seventh century when the Moslems overran North Africa and partly destroyed the Great Library of Alexandria. Part of the Library was preserved and taken to Constantinople by the fleeing teachers, and here another library existed until the fifteenth century.

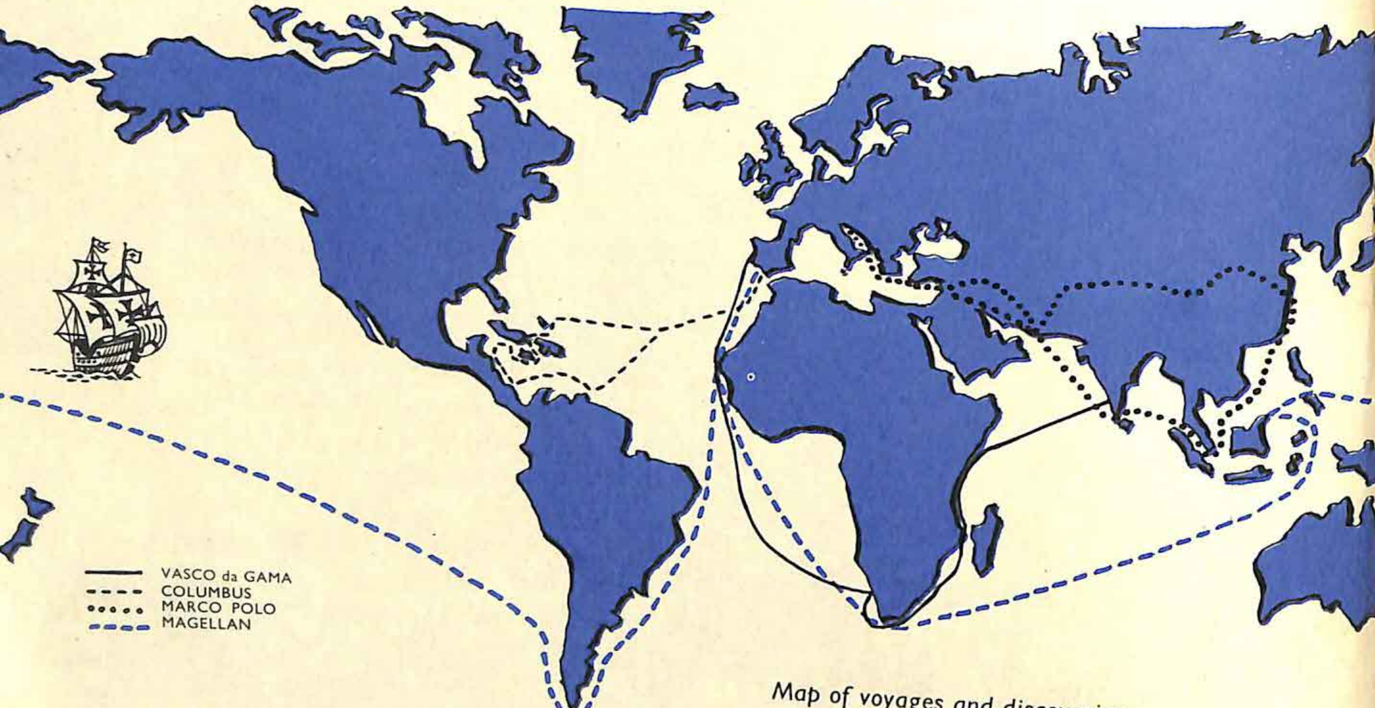
The Moslem empire spread from India through the Middle East countries to Spain. They became intrigued by Mathematics and admired the discoveries made by the Greeks and the Indians, but though they made use of it they did little more than preserve what they could after each of their battles was won. For centuries the Moslems remained custodians of Mathematical Science, discovering little but making many Arabic translations, and it was because of this that the early number systems trickled through Spain into the rest of Europe.

The growth of the Moslem Empire between 622 A.D. and 945 A.D.

The Crusaders returned from the Middle East with stories of many mathematical discoveries and Arabic manuscripts were translated into Latin. When Constantinople fell many of the scholars fled to the West and brought scrolls and knowledge from the Library at Constantinople.

At the beginning of the thirteenth century Universities were set up in Paris, Oxford, Cambridge, Padua and Naples. The students had a choice of two courses: (1) The Trivium, which consisted of Latin, rhetoric and logic and (2) The Quadrivium, including Arithmetic, Geometry, Music and Astronomy. Arithmetic consisted of keeping accounts for the monasteries while Geometry took in the first five propositions of Euclid. Music was related to the church services, and Astronomy involved such things as the calculation of Easter.

In the fourteenth and fifteenth century three inventions played a big part in reviving an interest in Mathematics. Gunpowder was made and this opened up the field of ballistics. The Chinese discovered a magnetic needle, which led to many voyages of discovery, and the invention of moveable type led to the publication of books and the standardisation of the number systems.



Map of voyages and discoveries.

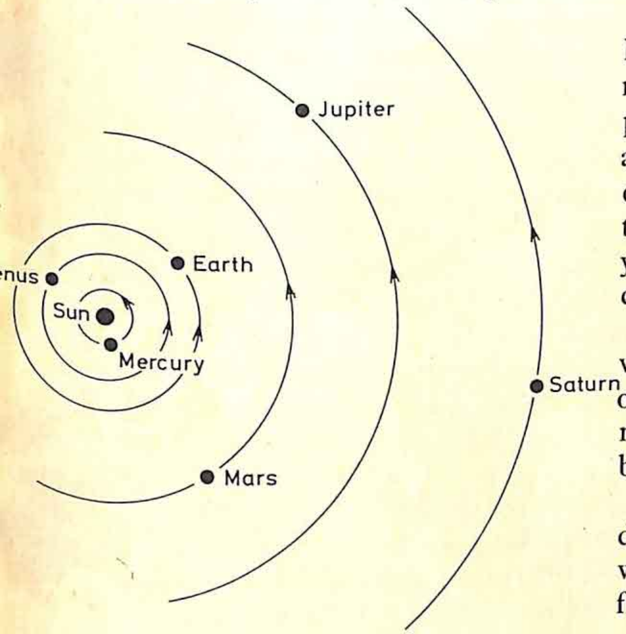
Up to this time people were so set in their beliefs that the earth was the centre of the Universe that to suggest anything else was considered sacrilegious, and disbelievers were often persecuted. One of the mathematicians of this time who had certain aspects of his work condemned was Copernicus.

COPERNICUS (1473–1543 A.D.)

Copernicus was educated at the University of Bologna where he studied Medicine, Astronomy and Mathematics. During his life he held a variety of jobs such as a bailiff, a tax collector, and he was even responsible for reforming the coinage in use at that time. Despite all this his main interest was Astronomy. His work was basically similar to some of the suggestions put forward by a Greek scholar named Aristarchus, who lived some three hundred years before the birth of Christ. Aristachus claimed that the earth moved around the sun and that the moon merely reflected light from the sun. Copernicus enlarged upon this theory but made the error of assuming the earth's path around the sun to be a circle.

In the face of great opposition from many influential people, he published

his famous theory with the sun at the centre of the universe and the earth and other heavenly bodies moving around it. This is known as the heliocentric system. During the course of his astronomical calculations, Copernicus encountered difficulties because the development of trigonometry had not kept pace with Astronomy. To overcome this he wrote a treatise on trigonometry which proved to be his only contribution to pure mathematics.



The opponents to his theories plied him with such questions as 'If the Earth revolves around the sun, why don't the positions of the stars appear to change as the earth travels in its orbit?' or 'Why does not the earth's motion cause things to fly off it at a tangent?' Can you with your present day knowledge answer these questions?

Copernicus wrote a book on his theories which was published in 1543, the year of his death, but opposition from the religious bodies of that time led to its being banned for a number of years.

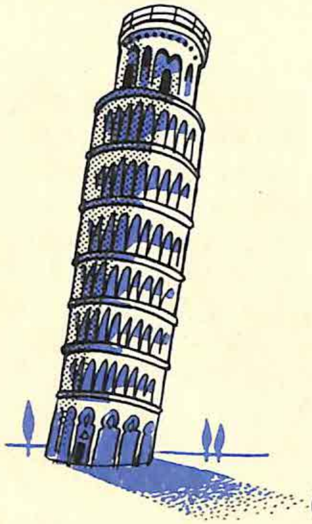
Another great mathematician w[illegible] lived during this time, when any for[illegible] ogress which was opposed to popula[illegible] s was frowned upon, was Galileo.

GALILEO (1564–1642 A.D.)

Galileo was born in Italy and his father was a poor nobleman. He eventually went to Pisa to study medicine, but one day as he sat in the Cathedral looking at the architecture, he was fascinated by the way in which the lamps swung to and fro. On timing them by using his pulse beat he discovered that the period of time for each swing or oscillation was constant.

From these simple observations a formula for finding the time of oscillation of a pendulum was discovered by Huygens, the Dutch scientist, in 1656. $T = 2\pi\sqrt{\frac{l}{g}}$ where T represents the time in seconds, l stands for the length of the pendulum and g is the acceleration due to gravity.

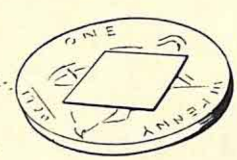
Galileo's experiments in what is now called the study of dynamics led to his becoming a professor at Pisa University. During this time he became interested in the effect of gravity on falling bodies, and a well known legend tells how he used the famous leaning tower of Pisa for a series of experiments.



The leaning tower of Pisa.

With an audience of students and professors from the University he dropped two stones from the top of the tower. One stone was ten times the weight of the other but they appeared to strike the ground simultaneously.

You can try a similar experiment yourself. Cut out a piece of paper about a half an inch square. If you release a penny and the paper simultaneously from the same height, the penny will reach the ground first. But according to Galileo they should both have fallen at the same rate. Why didn't they? The air resistance prevents this happening.



Try this experiment again placing the paper on top of the penny. You will see that they strike the ground at the same time, since the penny will be pushing away the air and the paper is subject to little or no resistance.

During his experiments Galileo discovered that a falling body increased its speed by 32 feet per second every second. He also discovered that the distance which an object fell was proportional to the square of the time taken for the fall. Thus evolved the formula $s = \frac{1}{2} gt^2$. Where s is the distance fallen, t is the time and g is the acceleration due to gravity.

Galileo's contemporaries were so horrified by this discovery, which contradicted the ancient laws laid down by Aristotle, that he was forced to resign his position at the university.

He moved to Padua where he was encouraged in his experiments and it is said that he needed a lecture room holding 2,000 students because of his popularity.

It was in 1609 that Galileo heard of the telescope which had been built in Holland. He saw the possibilities of such an instrument both in war and peace and demonstrated its use from the top of the highest church in Venice.

It is reported that ships were visible by means of his telescope some two hours before they could be observed by the naked eye, but Galileo was not satisfied

until he had made an instrument capable of making objects appear more than 30 times as large as they appeared to the naked eye.

From his observations of the stars, the moon and the sun, he confirmed the theory of Copernicus, and also that the moon obtained its light by reflections from the sun. He also discovered the four moons which orbit around Jupiter, the largest of all planets.

These ideas once again got him into serious trouble with the Church, as they did not coincide with the ideas that God had chosen the Earth as the most important part of the Universe. Once again he was forced to leave, this time he went to Florence where further confirmation of Copernicus' theories led to his being summoned before an Inquisition. He was made to promise, under the threat of torture, that he would not publish any more of his findings. He died in 1642 still under the supervision of the Inquisition.

Most of you will have seen a slide rule even if you cannot use one. The engineer and physicist make good use of it probably without realising how indebted they are to a Scots mathematician named Napier.

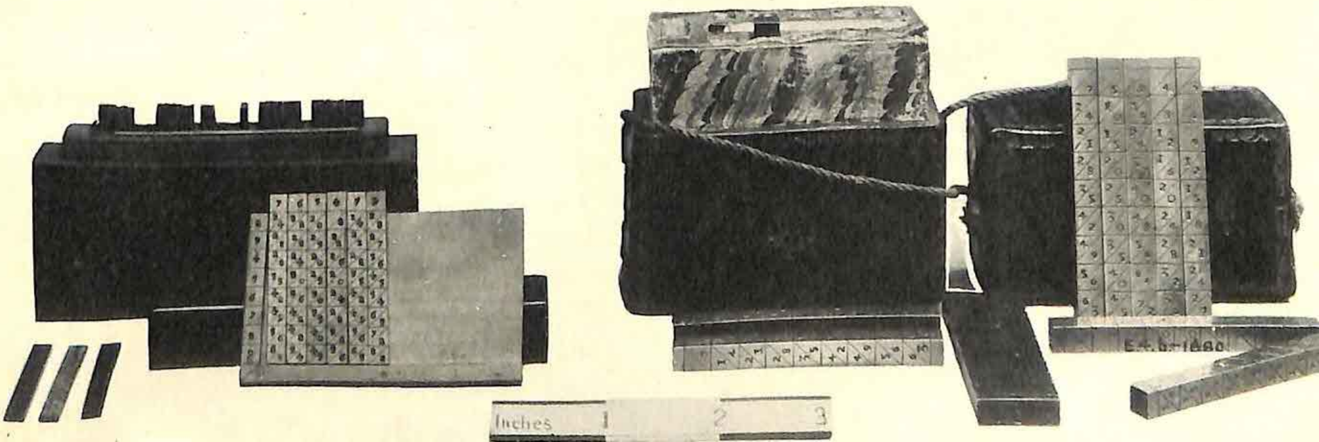
NAPIER (1550–1617)

Napier was born in Merchiston Castle at Edinburgh. At the age of thirteen his mother died and he was sent to St. Andrews' University where he eventually matriculated. He was then sent to study in Europe where at that time knowledge was regarded with far greater respect than in his native Scotland.

Napier became interested in finding a short cut through the tedious calculations which were linked with mathematics and astronomy in those days. One of his first inventions was a set of rods, known today as Napier's Bones. These were further developed into a cylindrical set.

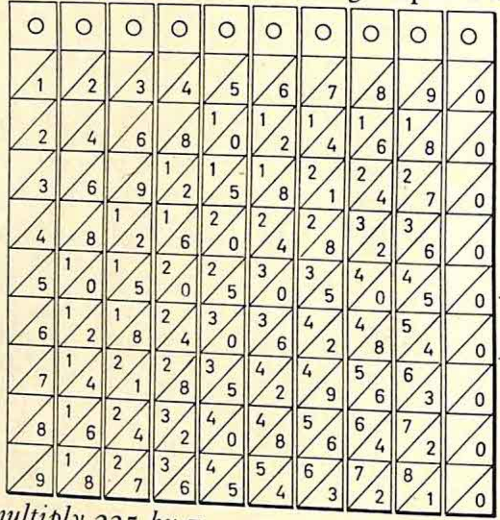
You can make your own set of rods to assist you in some of your calculations quite easily.

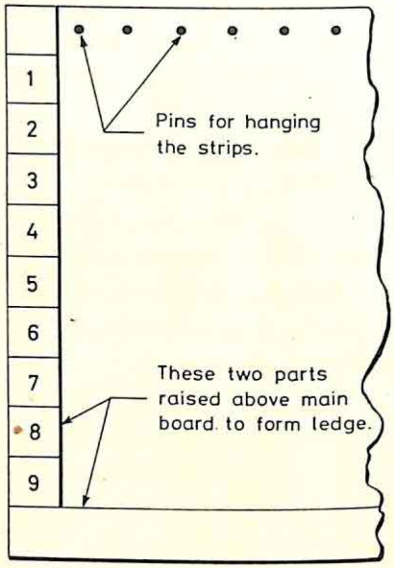
Cut strips of card each one half an inch wide and five inches long. Divide each strip into ten half inch squares and draw in the diagonal for each square. Pierce a hole in the top square of each strip and in the remaining squares write out your tables as shown in the illustration.



Make sure that you put the numbers in their correct positions on the squares. The base board is made by placing a row of pins along the top of the board at half inch intervals.

Now you are ready to start using Napier's Bones.





To multiply 235 *by* 7

Place strips 235 together on the base board. As you are multiplying by seven read your answer across the seventh row starting from right to left. Add up the numbers which appear in each parallelogram.

1 (4 + 2) (1 + 3) 5
1 6 4 5

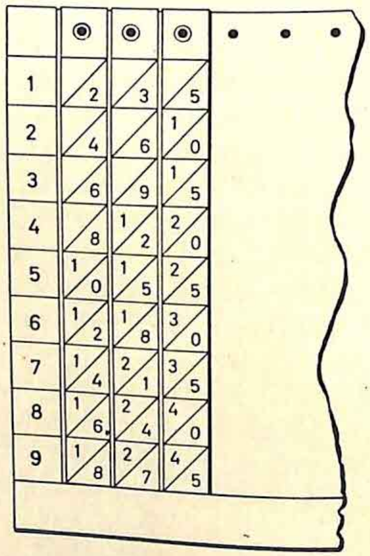
Therefore 235 × 7 = <u>1645</u>

To multiply 235 *by* 9

Read along the ninth row

1 (8 + 2) (7 + 4) 5
1 10 11 5
2 1 1 5

Therefore 235 × 9 = <u>2115</u>



Try and find a method of multiplying 235 by 28. You will soon discover that you often need a duplicate set of bones, for instance when multiplying 313 by 27 you would need two strips containing the three tables. This may be overcome by making your bones of balsa wood with a square cross section and fixing a different table to each face.

Napier's next step can be ranked as one of the most remarkable discoveries ever made. It is usual for discoveries to grow from ideas sown by earlier mathematicians. Napier's tables of logarithms took him almost twenty years to complete and were entirely independant of any mathematical knowledge available at that time. These tables reduced what had been tedious long multiplication and

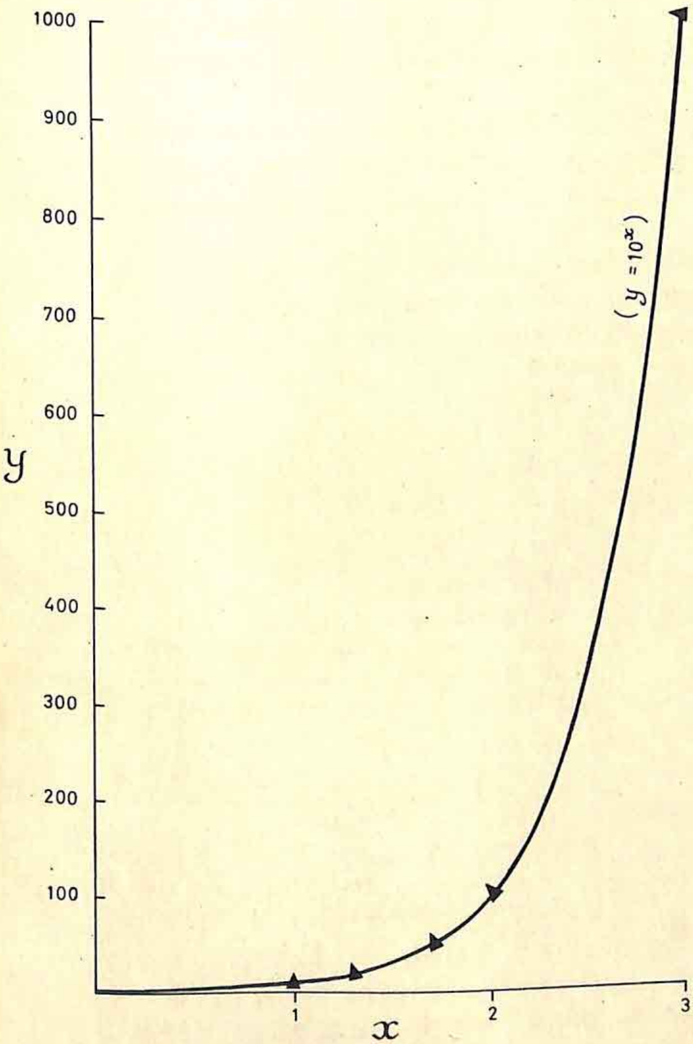
division to the simple process of addition and subtraction, a great assistance to the astronomer who was constantly having to use large numbers.

In Napier's second book which was revised by Briggs and published two years after his death, the whole numbers were separated from the fractions by a decimal point. The first set of logarithm tables to the base 10 was published by Briggs in 1617.

To make a log table to the base 10 draw a graph of $y = 10^x$.

From your graph you will see that
When y = 20   x = 1.3
When y = 50   x = 1.7

When x =	0	1	2	3
Then 10^x =	1	10	100	1000



Use your graph to complete this table.

y	x	y	x
10	1·0	100	
20	1·3	200	2·3
30		300	
40		400	
50	1·7	500	
60		600	2·78
70		700	
80		800	
90		900	

To multiply 20 × 50
When y = 20   x = 1.3
When y = 50   x = 1.7
Adding   x = 3.0

Now when x = 3.0   y = 1000
20 × 50 = 1000

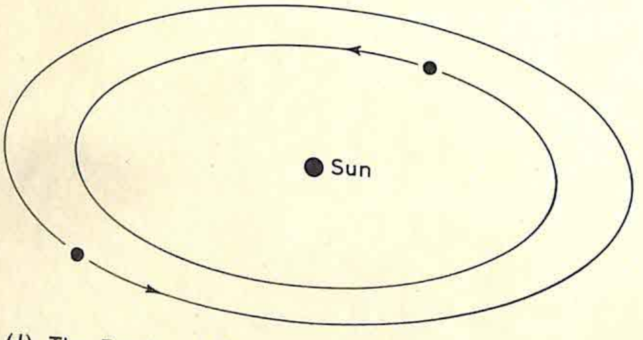
KEPLER (1571–1630)

Kepler was born near Stuttgart, and he was another of the mathematicians and scientists who continued his work despite opposition from his contemporaries. His father was not a rich man, but he managed to send him to school until he obtained a scholarship to a school for promising boys run by the Duke of Wurtenburg. Kepler went to the University of Tübingen to prepare for a life in the church, but he became interested in Astronomy, and was accepted as a lecturer at the University of Graz. Unfortunately the town came under the influence of certain religious bodies and Kepler was forced to resign from the position.

Soon after his expulsion he was introduced to the famous Scandinavian astronomer Tycho Brahe, who had spent twenty years of his life building up a comprehensive set of observations of the planets and the movement of stars. Before Brahe died he entrusted his knowledge to Kepler and asked him to complete the work.

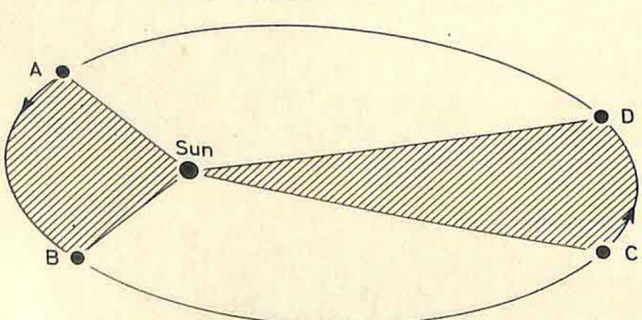
Kepler spent eight years compiling a book which dealt with the movement of the planet Mars. A great deal of his work was in vain until he discovered that the planet moved in an elliptical orbit and not a circular orbit as previously suggested by Copernicus.

This formed the basis for the first of his laws which are illustrated on this page. These laws did not only enrich the study of Astronomy, but were a basis for further work on conic sections, discovered centuries earlier by Apollonius.



(1) The Earth and the planets move in elliptical orbits around the Sun.

(2) A, B, C and D are different positions of the Earth in its orbit. When near to the Sun the planet moves quicker along its path.



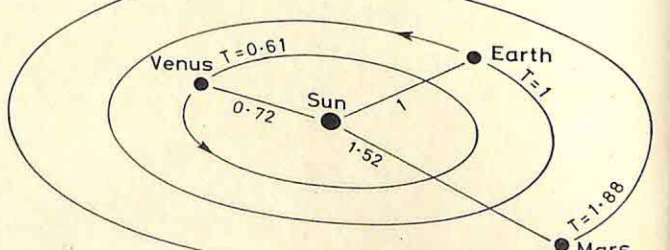
(3) If we let the year represent the unit of time, then Venus completes its orbit in 0·61 yrs. Mars completes its orbit in 1·88 yrs.

If we take the mean distance of the Earth from the Sun as 1 then the mean distance of Venus is 0·72, Mars is 1.88 Kepler's third law states that $\frac{T^2}{d^3} = K$

e.g.

Venus	Earth	Mars
$\frac{(0 \cdot 61)^2}{(0 \cdot 72)^3} = 0 \cdot 99$	$\frac{(1)^2}{(1)^3} = 1$	$\frac{(1 \cdot 88)^2}{(1 \cdot 52)^3} = 1$

This is true for all the planets.

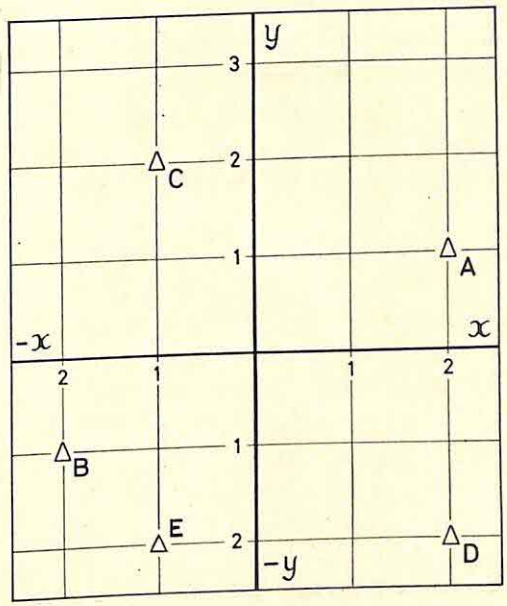


'I think; therefore I exist': these were the words of another mathematician and philosopher.

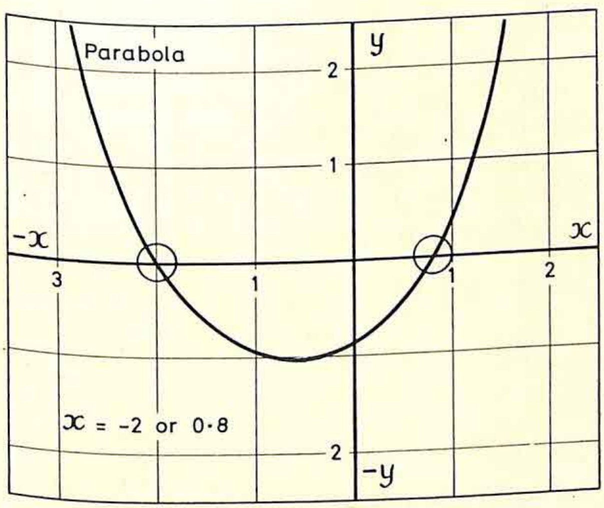
DESCARTES (1596–1650)

Often called the father of modern philosophy, he was a brilliant pupil. Descartes spent his childhood in a Jesuit school and at the age of 17 moved to Paris to continue his studies. He did however become very dissatisfied with the type of education he was following, so he gave it up and spent some years as a soldier in Holland, Bohemia and Hungary. In 1628 he returned to Holland where he lived for the next twenty years. Most of this time he spent studying and writing on philosophy, but on hearing of the fate of Galileo he decided not to have his works published until after his death.

There was only one small part of his writings which dealt with mathematics, but this was sufficient to mark him as an outstanding mathematician. Starting with Latitude and Longitude as a means of fixing any point on the earth's surface, he developed a similar system for use in graphical work.

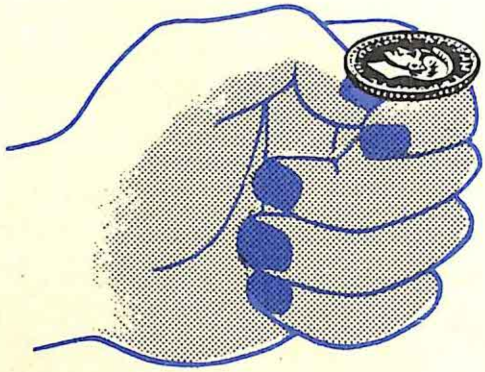


From the diagram point A would be described as (2, 1) i.e. $x = 2$, $y = 1$. Point B as $(-2, -1)$ i.e. $x = -2$, $y = -1$ Write down the description of points C, D and E in this form.



As his work developed Descartes found that by plotting a series of points, from a given equation, he could arrive at the curves previously known as the conic sections. Representing curves in this way, he also discovered a method of finding the values of x.

From 1629–1633 he prepared a work on philosophy, but knowing of the opposition to such works, and knowing the fate of Galileo, he decided to keep it secret. As a result his works were not published until after his death.



What are the chances of selecting the two of spades from a pack of cards?

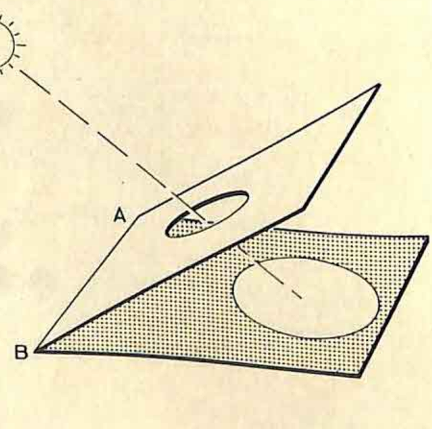
In solving problems such as illustrated above one must have some knowledge of the laws of probability; the foundations of such laws were laid by Blaise Pascal, a Frenchman.

PASCAL (1623–1662)

Because of his frail condition his father said that he should confine his studies to Latin and Greek and not tax his brain with Mathematics. This fact aroused Pascal's curiosity, 'What is this mysterious subject that people think is so difficult?' and he began to develop geometrical truths for himself. Pascal's father soon became aware of his son's intense interest in the subject and decided to foster this interest rather than subdue it.

By the age of 12 Pascal had mastered the first 32 propositions of Euclid and by the age of 16 he had written a complete treatise on conics.

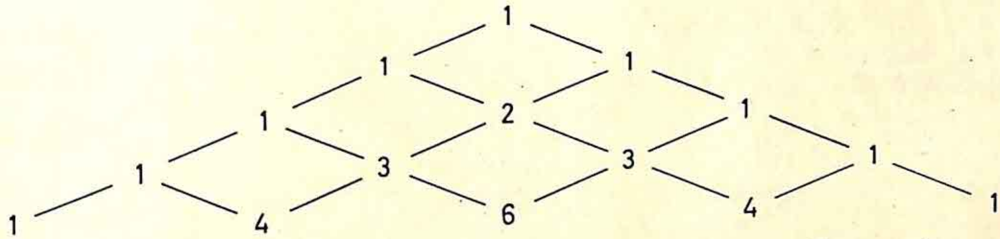
Cut a circular hole in a piece of paper and fold the paper along the line AB as shown in the diagram. When the paper is placed in the sunlight you will notice that the shadow projected on the lower half of the paper is in the shape of an ellipse. Shining a torch at an angle less than 90° to the floor will also give an ellipse.



Pascal was able to use similar methods of projection when studying the conic shapes.

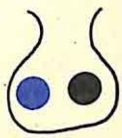
Gambling had become very popular at this time, and Pascal was asked to solve a problem which involved probability. The problem was solved independently by Pascal and Fermat, both of them using entirely different methods.

Pascal's method of solution was based on the triangular pattern which bears his name today.



A use of this triangle is illustrated in the following problem: From a bag containing two coloured counters, what are the chances of drawing one of the colours on four successive draws?

FIRST DRAW



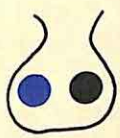
Assume that is drawn

SECOND DRAW If is drawn then the combination is

If is drawn then the combination is

If however the first draw had been different the combination would be as follows.

FIRST DRAW



Assume that is drawn

SECOND DRAW If is drawn then the combination is

If is drawn then the combination is

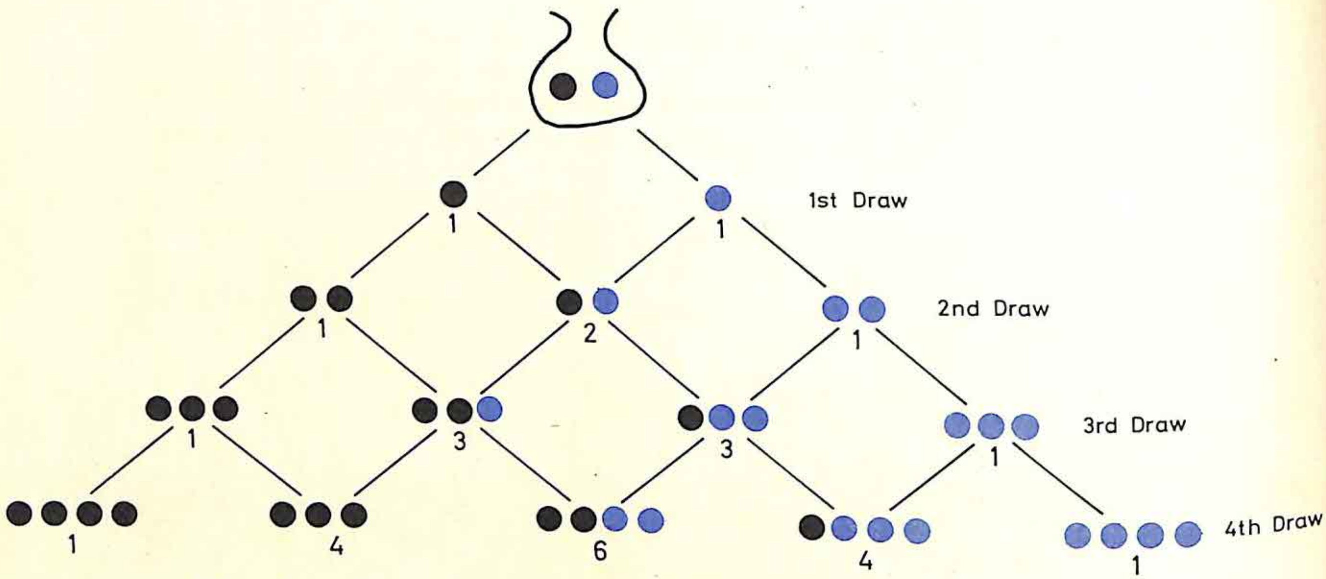
You will see that by making two successive draws you may arrive at any of the following combinations.

Notice that turns up once 1.

turns up twice. 2.

turns up once. 1.

The pattern 1, 2, 1 coincides with the relevant line in Pascal's triangle.



The above illustration shows the number of times which each combination can occur by making 1, 2, 3 and 4 successive draws from a bag containing two coloured counters.

From the sequence of figures shown for the fourth draw we can work out the chances of any combination happening: $1 + 4 + 6 + 4 + 1 = 16$.

Therefore the chances of withdrawing four BLACKS is 1 in 16. The chances of withdrawing two BLACKS and two BLUES are 6 in 16 and so on.

Similarly for the third draw, the chances of any combination happening: $1 + 3 + 3 + 1 = 8$. The chances of withdrawing three BLACKS are 1 in 8. The chances of withdrawing one BLACK and two BLUES are 3 in 8. Build up a similar diagram to illustrate the combinations of HEADS or TAILS obtained by repeatedly tossing a coin.

Probability plays a big part in our every day life, forming a basis for insurance tables, and statistical calculations among many other things, and, as you can see, it was by mere chance that investigations into these theories started.

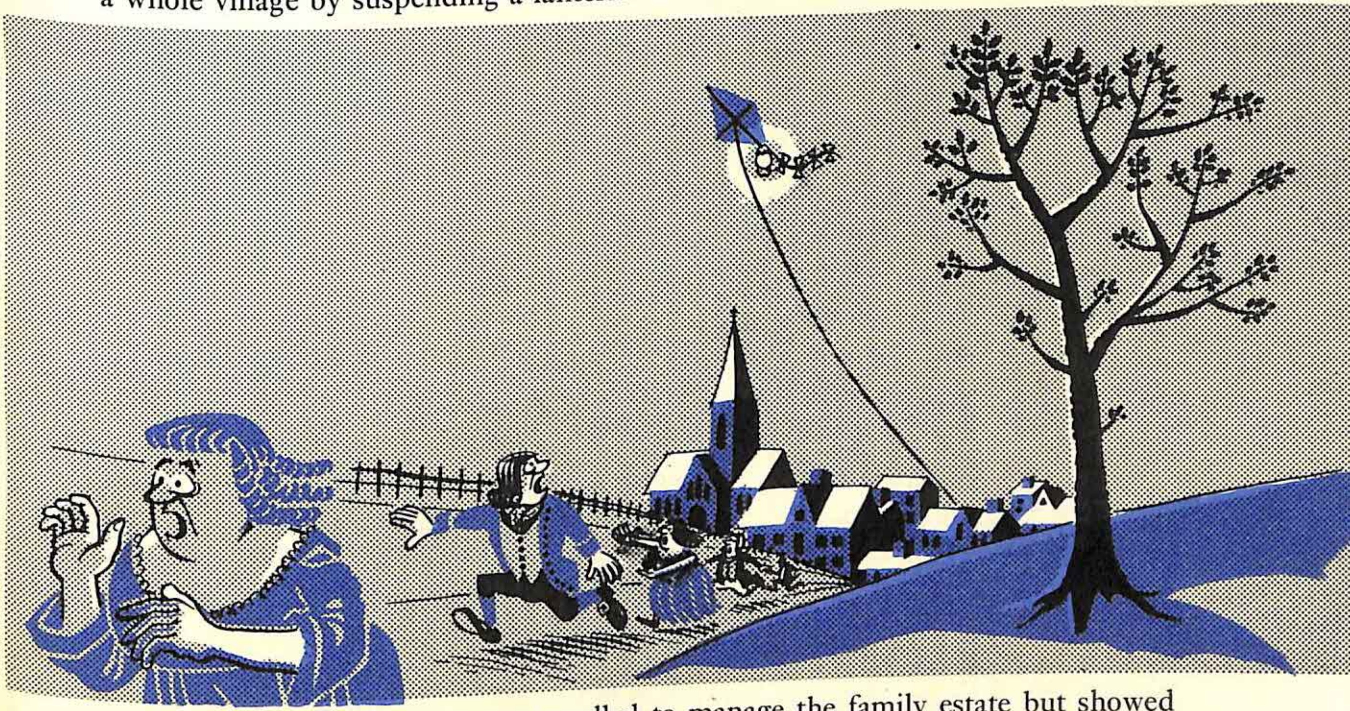
Travelling by horse and carriage, Pascal narrowly escaped death when the horses took fright and bolted. He thought it was by the Divine Right of God that he was not fatally injured. He was so convinced of this that he gave up mathematics and devoted his time to religion at which he worked relentlessly until he died at the age of 39.

NEWTON (1642–1727)

Newton was born in the small Lincolnshire village of Woolsthorpe near Colsterworth. At Grantham Grammar school he was an average boy, not very strong, and showing no sign of the hidden genius which was later to be revealed to the world.

As happens with most boys, however, he did one day become involved in a fight. Although Newton beat his opponent he was not satisfied until he was able to surpass the other boy in his academic studies as well. From this point onwards Newton's progress was so remarkable that he eventually became Head Boy of the school.

He did not join in games with other boys, but he did invent new games and he also became very interested in flying kites. There is a tale about him frightening a whole village by suspending a lantern from a kite at night.



At the age of 15 Newton was recalled to manage the family estate but showed so little interest that he was eventually allowed to return to his studies. When nineteen years old he went up to Cambridge. He knew very little geometry which was revealed to him when reading his first book, Keplers' 'Optics'. His immediate reaction was to buy and master a copy of Euclid's Elements.

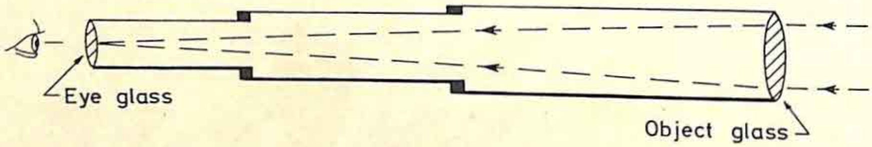
In 1665 the Great Plague created such a scare that the university was closed for two years. Unable to make use of the Library, Newton was forced to break off his experiments on light.

His active mind was soon at work again and during the next two years he not only invented a system of 'fluxions', which is now called Calculus, but he also began studying gravity.

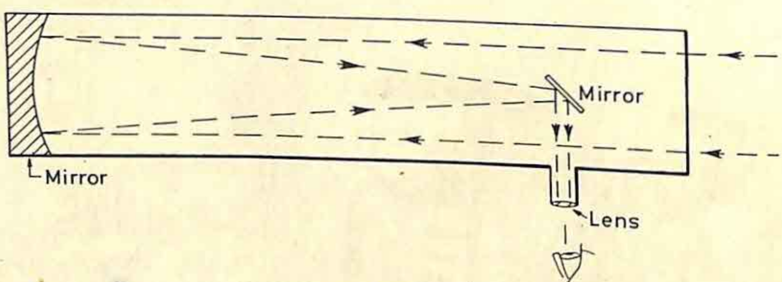
While sitting in the garden at Woolsthorpe he saw an apple fall from a tree and soon realised the truth in Galileo's experiments at Pisa.

Accepting these theories, he began to wonder why planets moved in elliptical paths round the sun instead of shooting off into space. He set down a series of calculations in an attempt to find a mathematical explanation for this phenomena; just as the ancient Greeks had done when confronted by their problems. He, unfortunately, took the earth's radius as 3,440 miles instead of 3,960 miles. This mistake caused him to abandon this line of research for almost 16 years.

Newton was not satisfied with the blurred image he obtained when using a refracting telescope and set about trying to improve it. During his investigations Newton discovered that the blurred image was due to the dispersion of light His efforts to correct this led to the invention of the reflecting telescope which he constructed at Cambridge in 1668.



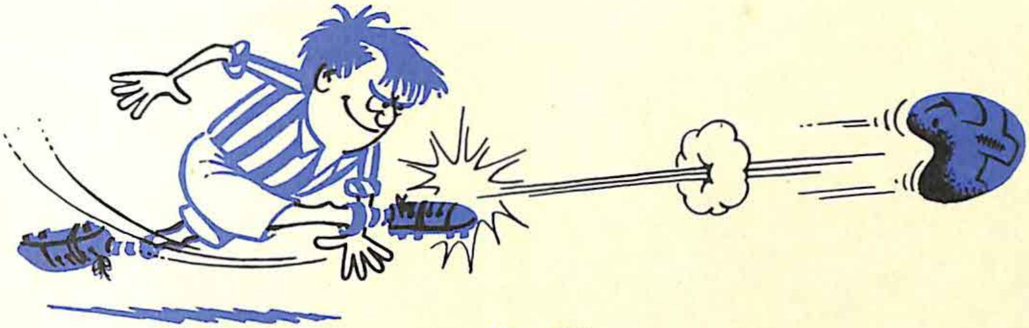
Refracting Telescope



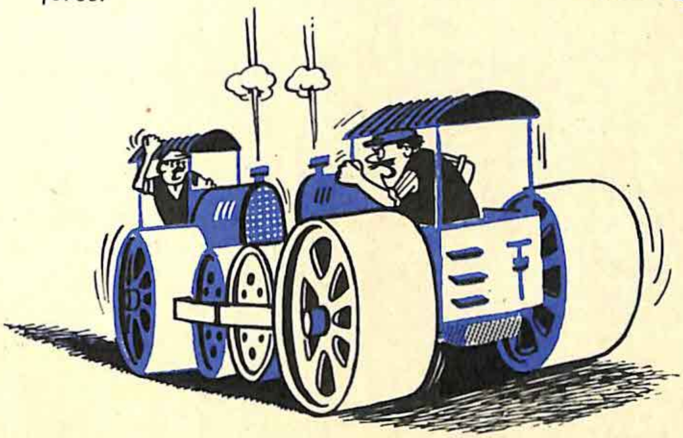
Reflecting Telescope

Once more Newton had the University Library at his disposal. Returning to his study of the planets, he realised that his previous calculations had been wrong. The error was due to his using the wrong radius for the earth. As ever, Newton was very slow in publishing his works and it was fortunate that he had two good friends in Wren and Halley. (Wren designed St. Paul's cathedral and other important buildings built after the Fire of London. Halley, an astronomer, discovered a comet which bears his name today.) Both of these great men were interested in Newton's work, and it was due to pressure from them that he finally published Principia, his greatest work.

Among the discoveries explained in his book were theories relating space and time, his three laws of motion, movement of planetary bodies, the motion of the pendulum and the tides.

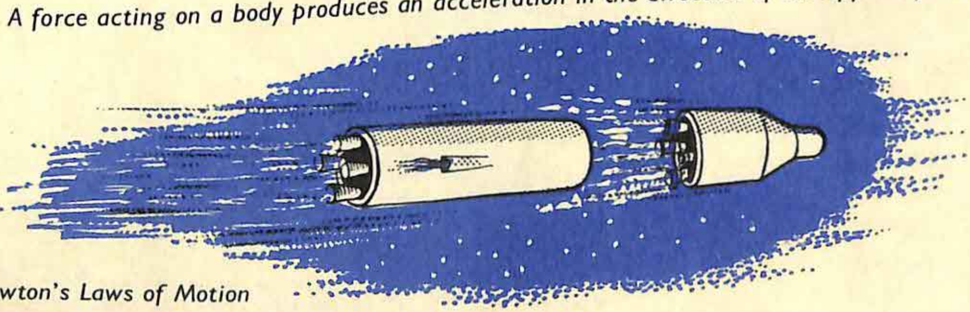


(*a*) *A body at rest remains at rest and a body in motion continues to move at constant speed in a given direction, unless acted upon by an external force.*



(*b*) *To every action there is an opposite and equal reaction.*

(*c*) *A force acting on a body produces an acceleration in the direction of the applied force.*



Newton's Laws of Motion

Newton did not make any outstanding contributions to mathematics in the later years of his life.

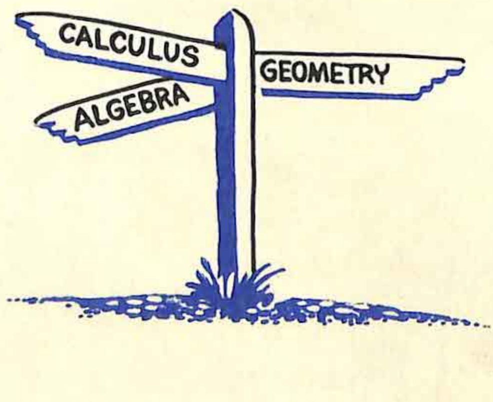
By 1696 the coinage system in England was very bad. The original silver coins had been replaced by soft alloy coins which were cheaper to produce. Newton undertook the task of rectifying this fault.

During his life time Newton was involved in one of the most bitter quarrels during the whole history of mathematics. He made his discovery of Calculus (which he called fluxions) in 1666, but he did not publish his findings until 21 years later. By this time a German mathematician had made an almost identical discovery which he published in 1684. But the approach of the two men to the 'Calculus' was quite different. The name of this mathematician was Leibniz.

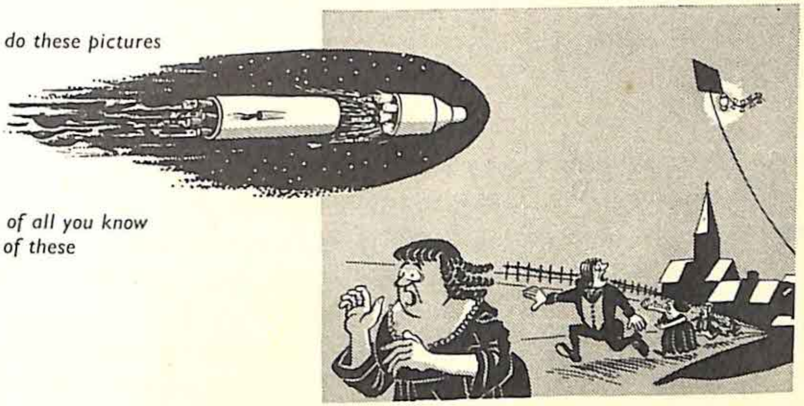
LEIBNIZ (1646–1716)

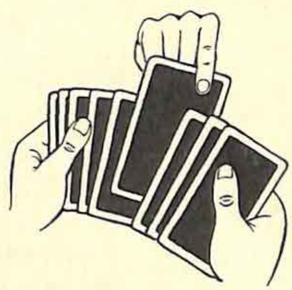
Leibniz was recognised as being a brilliant student. It is generally accepted that his Calculus was much clearer and far more detailed than that of Newton. Each man claimed that he had invented the calculus. From this time the continental mathematicians accepted the teachings of Leibniz, and ignored Newton's findings, while in England the reverse of this took place. It was more than a century before the two groups came together, exchanging and accepting one another's ideas freely.

You will realise from what has been explained that most of the Mathematics included in the Secondary course was known by the 17th century. But with the invention of calculus, mathematics began to develop rapidly. It is said that Henri Poincaré (d. 1904) was the last mathematician to be able to handle the whole field of Mathematics. Since his time the subject has spread and become highly specialised, some parts becoming so abstruse that it is hard to believe that they will ever have any practical use.

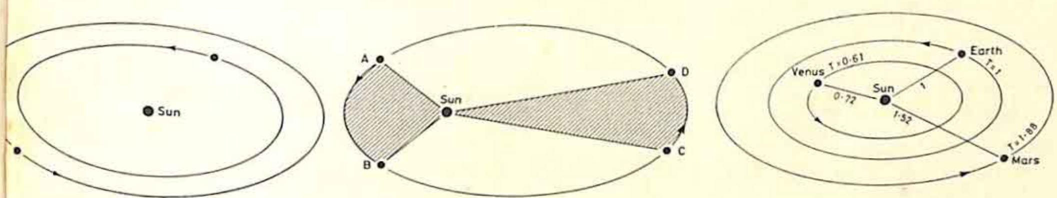


Of which mathematician do these pictures remind you?

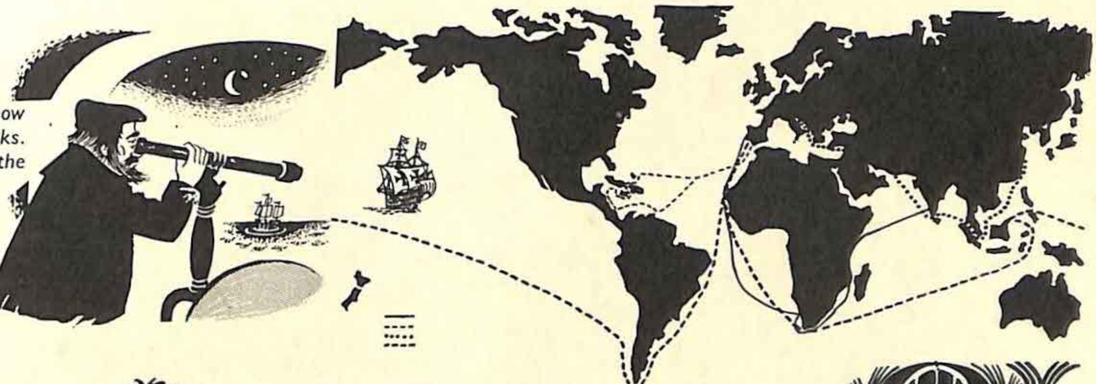


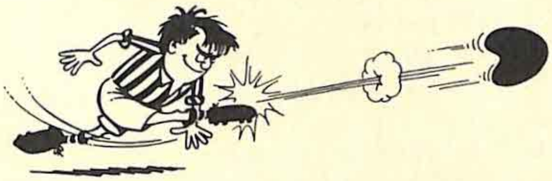


Write a brief description of all you know about the lives and work of these mathematicians.



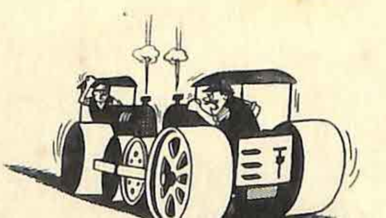
Make a drawing to show how a refractor telescope works. How does this differ from the reflecting telescope?

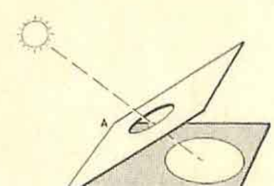


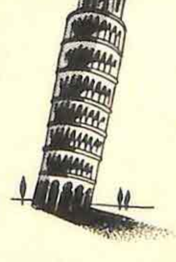


Can you say which laws are illustrated here?

How did Galileo measure the time taken for the lamp to swing to and fro?







MONEY

The term money conjures up a picture of coins and notes which we carry about and exchange for food, clothes or whatever commodities we may require. Children sometimes resort to a much simpler and older method called barter, which dates back to the time when primitive man realised that if he had more food than he needed he could often exchange it for some other article which he wanted. The Ancient Egyptians paid their rent and taxes by surrendering part of their crops. Explorers in Africa used to carry brightly coloured materials and beads to exchange with the natives for food. The Maoris bartered large areas of New Zealand for blankets and guns. You will realise that barter is not always a fair means of exchange and so traders began accepting precious metals or stones as payment. The Spartans believed that wealth was the root of all evil. The iron coins which they used were so large and heavy that it was very inconvenient to carry or store large fortunes.

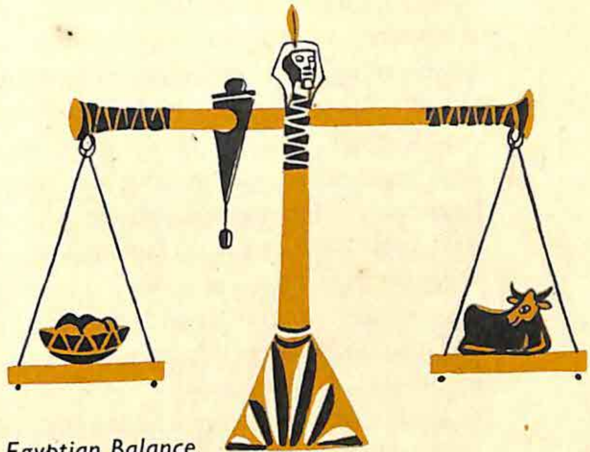
When North America was explored the Indians had already invented a primitive coinage system. They collected shells of a certain shape, strung them like beads and called them wampum. The beads were coloured and their value was based on their colour. One purple shell might have been equivalent to two white ones.

When the Romans came to Britain they found that rings of brass, silver and gold were in use. They introduced their own system and used the terms Libra, Sestertius and Denarius which were their names for gold, silver and bronze coins used at that time, the letters £. s. d. remain with us to this day.

Coins have been found which were in use as early as the 7th century B.C. in China and the Mediterranean countries. Relics of the 4th century B.C. were stamped with pictures of an ox or whatever animal or article they represented. The Tower pound, so called because it was minted in the Tower of London, was the value of one pound weight of silver. A pound's worth of present day silver coins only weigh about 4 ozs. Gold Sovereigns were still used for the pound up to 1918, when paper money was brought into circulation and in 1951 gold coins were taken out of circulation. Other coins which have been substituted or withdrawn include the five shilling piece or crown, the groat which was worth fourpence and the silver penny. The silver threepenny piece was replaced by a larger twelve sided coin in 1937 and in 1961 the farthing, which had become obsolete was withdrawn from circulation after being in use since the days of Charles II.

WEIGHTS

Pictures of simple weighing scales or balances, dating back to 3000 B.C., have been found on Egyptian papyrus and on the walls of an Egyptian temple. The weights in use then were made of stone and shaped like the object being weighed. One of these weights was shaped like an ox and was called the Talent and it is interesting to note that the weight was linked with a measure of volume. It is claimed that the Talent weighed the same as the Egyptian Royal Cubic foot of water.

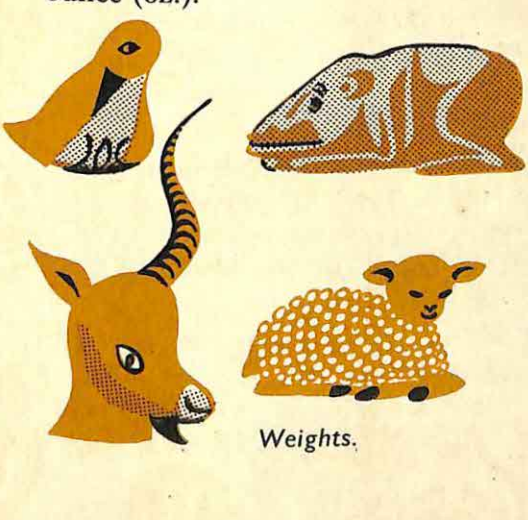

Egyptian Balance.

1 Egyptian Royal foot = 13,76 inches
Therefore 1 cubic foot = $(13.76)^3$ cu. ins.
Now 1 cu. ft. of water = $62\frac{1}{2}$ lbs.
Therefore 1 Egyptian Royal Cubic foot of water weighs 94.43 lbs.

The talent was a fairly large unit and so it was divided into 3000 parts. Each part was called a Shekel and this weighed slightly less than the modern half ounce.

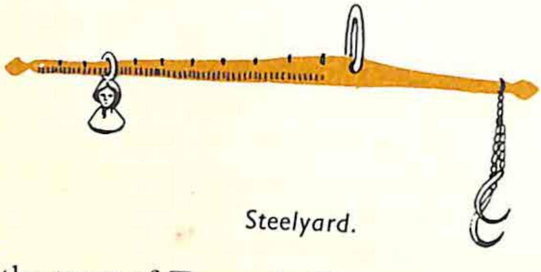
The Bible makes numerous references to the talent and the shekel, for example Goliath's coat of mail was made from 5,000 shekels of brass.

Other systems of weights and measures had been built up but there was no means of standardisation by this time. The Romans traded with the Egyptians and came up against these difficulties. As a result they then adopted some of the Egyptian standards and modified others to suit their own purpose. At one time the talent was divided into 125 parts, each of which was called a libra and it is from this that the abbreviation lb. is derived. The libra was divided into 12 parts, each called an uncia. This term later became onza and so today we have ounce (oz.).


Weights.

The Romans also linked their weights with measures of volume. They used a stone topped table in which they fixed bowls so that it looked something like a washbasin. These were known as the mensaponderaria, or measurements of weight.

In the Far East certain dried seeds were used as standard weights and a similar system was in operation in England. This was replaced by the introduction of the Troy system of weights in the latter part of the 13th century. The name



Steelyard.

was derived from the town of Troyes in France. In this system 24 grains of dry round wheat make 1 pennyweight and 20 pence make one ounce, while there are only 12 ounces in the pound Troy. These are the weights used by goldsmiths today.

For other goods we use the pound avoirdupois. This is heavier than the pound troy by 1,240 grains. One sixteenth of a pound is one ounce while a hundredweight is 100 lb. with 12 given for good measure.

Until 1427 there were no standard weights, merchants using whatever weight they chose. They often used one weight when buying their merchandise and another when they were selling to a customer. At this time a set of standards was kept at Oxford University so that the merchants' weights could be tested. This was the beginning of the British Standards units which are used today.

Buying.

Selling.

In the reign of Queen Elizabeth I (1542) a committee was formed to draw up standard tables. But some merchants continued to use Troy weights while others used avoirdupois. It was eventually decreed in 1587 that Troy weights would be used for bread, gold and silver only, specimen weights being sent to the various towns and cities.

In 1847 a Royal Commission recommended a new system of standards which were kept by the Board of Trade. This practice continues today and inspectors travel around the country checking scales to see that measures are accurately maintained.

LENGTH

You have read how the Nile floods fertilised the land and made it possible for town and cities to grow along this narrow strip through the desert. The Egyptians paid rent for their land, and because the river changed its course over the years it became necessary for them to have some means of measuring. We are not sure how the first measurement began, but some form of early measurement is mentioned in the Rhind papyrus kept in the British Museum, and by the time the Pyramids were built, considerable advances had been made.

Most of the standards used were taken from the body, and some of these are in use up to the present time though their names and values may have changed. One example of this is the Royal Cubit used in the construction of the Pyramids which was 20.62 ins. as opposed to 18 ins. which is the average cubit of man.

4 DIGITS = 1 PALM
3 PALMS = 1 SPAN
2 SPANS = 1 CUBIT
4 CUBITS = 1 FATHOM

Greater distances were measured by paces and by 280 B.C. the Romans were measuring distances with the double pace which is approximately 5 feet. They called this the passus, 125 of which gave 1 stadium and 8 stadia were known as 'mille passus' (1000 paces). This is the origin of our mile.

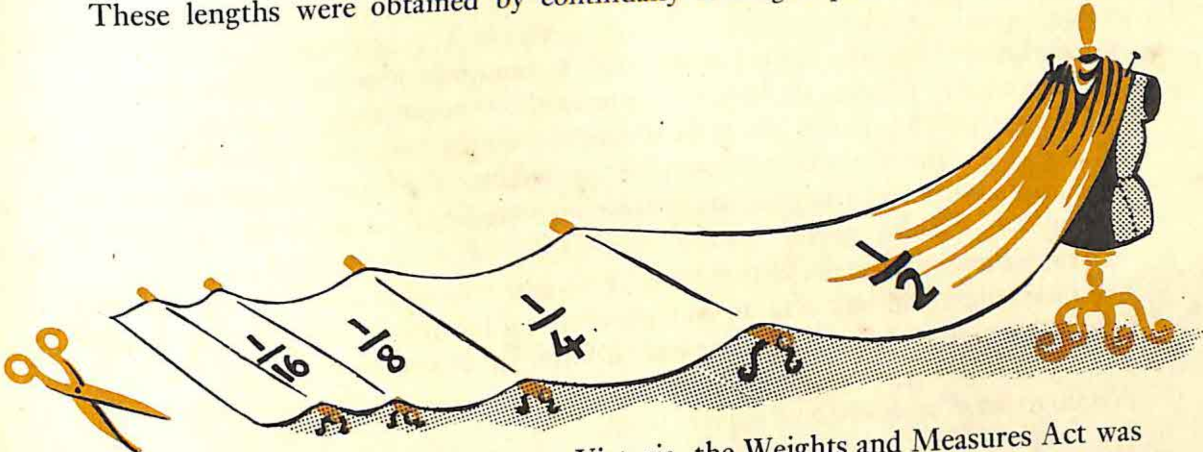
As with weights it was possible for merchants to use a long 'yardstick' when buying cloth and a short 'yardstick' when selling. The first law concerning the measurements taken from the body were abolished and three barley grains were accepted as 1 inch.

Legend tells us that in Henry I's time several men coming out of church were lined up and had their feet measured, the average being taken as the English foot of 12 inches. It is also suggested that the yard was the length of Henry I's arm, from the tip of his middle finger to his nose. I am sure that you have seen people using such a method today.

Measuring the English foot.

During the reign of Edward III it was ordained and standard measure should be used throughout the land; but it was not until the time of Henry VII in 1496 that a brass standard yard was made. New standards were made in the reign of Elizabeth I, marked on one side in feet and inches and on the other side in fractions ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and $\frac{1}{16}$) of a yard.

These lengths were obtained by *continually* folding a piece of cloth in half.



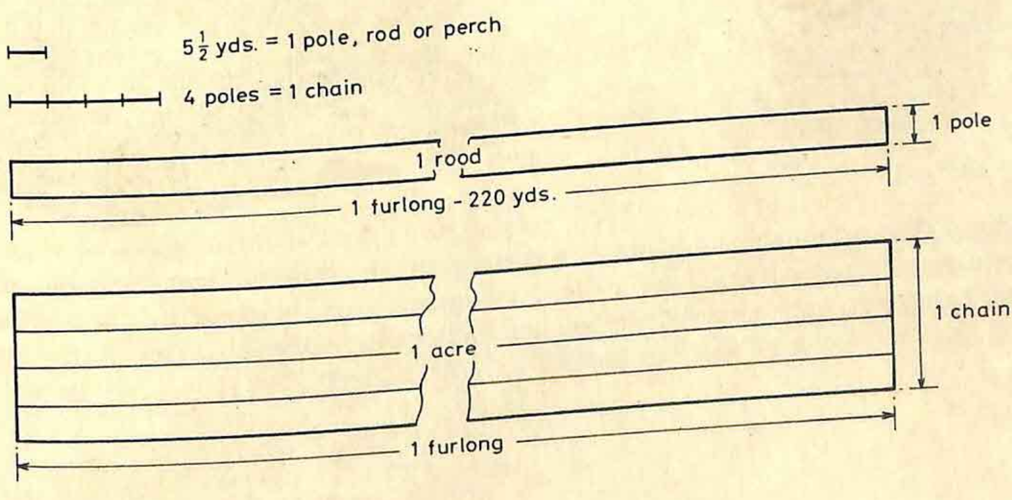
In 1878, during the reign of Queen Victoria, the Weights and Measures Act was passed by Parliament, stating that the standard yard should be the distance between two marks on a bar of metal kept in the standards office in London. Two small gold plugs were let into the bar and the marks were cut in these.

The standard yard in current use was made to replace the one destroyed by bombing in the last war and is placed in the Guildhall in London.

MEASUREMENT OF LAND

For centuries the land was ploughed by a team of oxen. The farmer urged his beasts to greater effort by means of a long pole and ploughed the length of his field in a furrow. At the end of each furrow the oxen had to be turned around ready for the return journey. The furrow was made about $\frac{1}{8}$ of a mile wherever possible, and this became known as the furlong—220 yards. The stick which was used to drive the oxen was called the rod, pole or perch and was about $5\frac{1}{2}$ yards long, a measure not often used today.

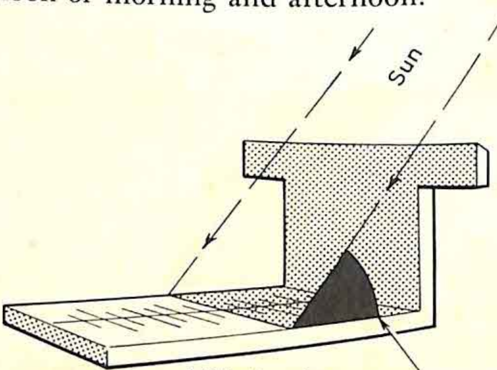
From these two measurements you can see how other land measurements have developed.



TIME

The earliest way in which man recorded the passing of time was by the counting of the periods of light and darkness. He realised the importance of the sun, and all over the world evidence has been found to show that early man worshipped the sun as a God. The Egyptians called their sun-god Ra, while the Greeks used the name Helias and the Romans worshipped Apollo.

The Persian day began at dawn while the Greek day was from sun set to sun set. Ptolemy eventually decided that the day began and ended at noon, which was the time when the sun was at its highest point overhead and cast the shortest shadow. Thus we have the two divisions, forenoon or morning and afternoon.



This interest in shadows led to the invention of the first clock about 1500 B.C. which was merely a device for measuring shadows. The period of daylight was divided into twelve hours, which differs from our present system because the period of sunlight varies throughout the year. In Egypt which is fairly close to the Equator, this period varies from 10 to 14 hours, while in countries further north such as Britain, it varies from $7\frac{3}{4}$ hours to $16\frac{1}{2}$ hours. This gave rise to the situation where a period during the day could be 50 minutes while a period of time at night could be 1 hour 10 minutes.

This fact was soon realised and the Egyptian water clock shown in the illustration allowed for this. Shaped like a flower pot, it was graduated on the inside and had a different scale for each month of the year.

Egyptian Water Clock.

Roman Sundial.

Sun dials and water clocks were developed by the Romans and their use was common in all parts of the Roman Empire. One such sun dial was made by hollowing a hemi-sphere out of a flat slab of stone. A style was placed in the hemisphere equal to the depth of the hemisphere. This style cast its shadow in the bowl

and by following the tip of this shadow a curve was formed. This type was later cut in half and an example has been found which dates back to 300 B.C. Once again the curve was divided into twelve parts which varied from season to season.

After the decline of the Roman rule in Britain about 450 A.D. King Alfred used candles to record time. Each candle burned out in 4 hours and to prevent draughts from affecting it a wooden or horn lantern was used.

In Saxon times another type of water clock was used. A bowl of about 10 inches diameter had a hole pierced in the bottom and it was floated on water. The water gradually found its way into the bowl until it finally sank, the time taken for the bowl to sink being taken as the period of time.

It was during this period that sun dials were placed on the sides of churches and some of these still exist today. They usually consisted of a slab of stone placed in the South wall of the church and engraved with marks to show certain times. It has been suggested that these marks were sometimes used to show the times of church services, while others divided the day into four parts or tides. This may well be the source of the terms 'eventide' and 'noontide'.

From the 12th to 15th centuries sundials made of metal and divided into hours were set up in many public places, and it was also during this time that the idea of having equal hours was firmly established.

The dividing of each day into 24 equal hours led to a great deal of progress in the construction of mechanical clocks of all kinds. Some depended on a pendulum swinging to and fro while others had a spring which was wound up and then allowed to unwind itself. Other clocks were designed to work from atmospheric

pressure and in the present century we make use of electricity.

The latest invention is the atomic clock designed at the National Physical Laboratory which is claimed to gain or lose only 1 second in 1,000 years.

Clocks through the ages.

THE STORY OF THE CALENDAR

The Priests of Ancient Egypt had divided their year into the Water season, the Garden season and the Fruit season. By counting the days and keeping watch on a set of poles, called Nilometers, which recorded the level of the river, they impressed the ordinary people by forecasting the floods accurately.

This method of counting days was greatly simplified when it was observed that the moon underwent a complete cycle in approximately 30 days.

The days of the week are derived from the Roman method of allowing certain planets or heavenly bodies to rule for one hour each day, beginning with the first hour on Saturday.

Romulus and his brother Remus were reputed to have been fed and looked after by a wild shewolf. They were the founders of Rome and Romulus devised a calendar in which a year of 304 days was divided into 10 lunar months, 6 having 30 days and 4 having 31 days. The first month he named March after his father Mars, who was also the Roman God of War.

SATURN	JUPITER	MARS	SUN	VENUS	MERCURY	MOON
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	1	2	3	4	5
						etc.

Saturday, Sunday, Monday, Tuesday

The second King of Rome was Numa Pomphilius and he modified this calendar by adding the months January and February. This gave him a total of 354 days but because he believed even numbers were unlucky an extra day was included. Nothing was done about the names of the months and even today we have September (seventh month) representing the ninth month of the year. By looking at the calendar you will be able to find several other months whose names do not make sense. Numa realised that his calendar was 10 or eleven days short and to overcome this he included an extra month every other year.

By 46 A.D. Julius Caesar found that the days were almost three months out of place and he decreed that the year 46 A.D. should contain 455 days. You will see that the Julian calendar which he drew up is very similar to the one we use today. The seventh month he named after himself.

JULIAN CALENDAR		
JANUARY	31	31
FEBRUARY	29	30
MARCH	31	31
APRIL	30	30
MAY	31	31
JUNE	30	30
JULY	31	31
SEXTILIS	30	30
SEPTEMBER	31	31
OCTOBER	30	30
NOVEMBER	31	31
DECEMBER	30	30
	365 ORDINARY YEAR	366 LEAP YEAR

Very few changes were made and this calendar was in use for nearly 2,000 years.

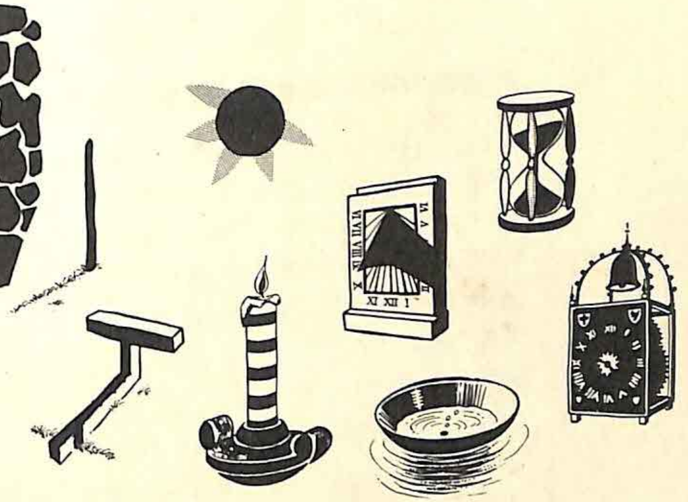
One of these changes was made by Augustus Caesar, who decided to name the eighth month after himself and also he wanted the same number of days in August as in July. This led to the calendar which we use today. It also gives the rhyme which many of us use:

Thirty days hath September,
April, June and November,
All the rest have thirty one
Except February alone
Which has twenty eight days clear,
And twenty nine days in a leap year.

In the Julian Calendar a leap year occurred every four years, but in 1452 the Gregorian Calendar was introduced, which allowed a leap year every four years except at the beginning of a century when no leap year was included unless the date was divisible by 400.

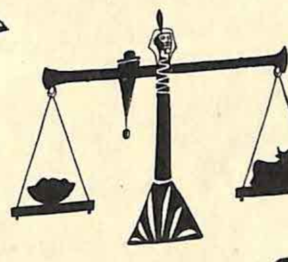
The Gregorian Calendar was introduced into Britain in 1752 when it was found that the Julian Calendar was 11 days different from the true calendar. The Government decided that the 3rd September should become the 14th September. Opponents of this change spread rumours that the people were being robbed of 11 days and this led to riots and demonstrations demanding the return of the 11 stolen days.

The end of the old calendar did not come until 1918 when America and Russia finally changed over completely to the new calendar.

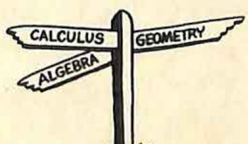


Give a brief description of the development of time.
Name the different time pieces in the illustration.
Can you explain how they worked?

Name the different parts of the body which have been used as a unit of measurement.



Can you name some of these weights and say who used them?



How is a steelyard used?

Explain why these riots took place.

GLOSSARY AND INDEX

ABSTRACT NUMBERS. Numbers which do not refer to objects, i.e. numbers used on their own. 7

ACCELERATION. Rate of change of velocity or speed. 35, 47

APEX. Top, corner or point of a triangle, pyramid or cone. 16

ARITHMETIC. Study of the behaviour of numbers. 33

ASTRONOMY. Study of the behaviour of planets and stars. 18, 34

BALLISTICS. Study of the behaviour of projectiles i.e. bullets, rockets. 34

BISECT. To cut into two equal parts. 15

CARDINAL POINTS. Four most important points of a Compass. North, South, East, West. 15

CIRCUMFERENCE. Distance around the outside of a circle. 14, 29

CONCENTRIC CIRCLES. Circles of various diameters having the same centre. 15

CONIC SECTIONS. Shapes obtained by cutting a cone with a plane surface: circle, ellipse, parabola, hyperbola. 30, 40

DECIMAL SYSTEM. System of counting using 10 as a base. 18

DIAMETER. Straight line through the centre joining two points on the circumference of a circle. 57

DODECAHEDRON. One of the five regular solids found by Pythagoreans. It has twelve faces each of which is a regular pentagon. 20

DYNAMICS. Study of the action of forces on a moving body. 36

ECLIPSE. (Solar) When the moon comes between the Sun and the earth and so obscures the sun's rays. 18

ELLIPSE. Oval shape which may be made by cutting a cone with a plane surface. 30, 42

GEOMETRY. Study of lines, surfaces and solids. 22

HELIOCENTRIC. Taking the sun as the centre of the Universe. 35

HELICAL. Spiral. 27

HYPERBOLA. Curve produced when cone is cut parallel to the vertical axis. 30

HYPOTENUSE. Longest side of a right angled triangle/or side opposite right angle. 22

ICOSAHEDRON. One of the five regular solids found by Pythagoreans. It has twenty faces each of which is an equilateral triangle. 20

LATITUDE. Measures position of a point North or South of the Equator. 41

LONGITUDE. Measures position of a point East or West of Greenwich. 41

OBELISK. Tall square stone pillar with a pointed top. 15

OCTAHEDRON. One of the five regular solids found by the Pythagoreans. It has eight faces each of which is an equilateral triangle. 20

ORIENTATE. To fix the position of an object. 15

OSCILLATE. Swing to and fro. 35

PARABOLA. Curve produced by cutting a cone along a plane parallel to its sloping side. 30

PARALLELOGRAM. Four sided figure having opposite sides parallel. 25, 38

PENTAGON. Five sided figure. 20

PLANE. Flat surface. 30

PROBABILITY. Likelihood of an event occurring. 44

RADIUS. Distance from centre of a circle to the circumference. 14

SIMILAR. Same shape but not always the same size. 8

SOLSTICE. Time when the sun is furthest North or South of the Equator. 29

STADIUM. Measure of length (approximately 202 yards). 29, 54

STATICS. Study of forces in bodies at rest. 27

STATISTICS. Collecting together of numerical facts. 44

TANGENT. Line which touches a curve at one point only. 35

TETRAHEDRON. Pyramid on triangular base. 20

TRIANGLE. Shape made up of three sides and three angles. Equilateral: Sides and angles are all equal. Isosceles. Two sides and two angles are equal. Right Angled. One of the angles is 90°. Scalene. All angles and sides are different. 5, 14, 19

TRISECT. Cut into three equal parts. 24

Many volumes have been written about the History of Mathematics and the few listed below will interest the reader who wishes to extend his knowledge beyond the limits of this book.

Title	*Author*	PUBLISHER
Stories of Mathematics	S. Ewart Williams	Evans
Man Must Measure	Lancelot Hogben	Rathbone Books
Mathematics in the Making	Lancelot Hogben	McDonald
The Pyramids of Egypt	I. E. S. Edwards	Pelican
Makers of Mathematics	Alfred Hooper	Faber
The Great Mathematicians	H. W. Turnbull	Methuen
A Short History of Mathematics	Vera Sanford	Houghton Miflin and Co.
History of Mathematics	David Eugene Smith	Ginn and Co.
Discovering Mathematics	H. A. Shaw and F. E. Wright	Edward Arnold
The Story of Measurement	Thyra Smith	Blackwell.

ACKNOWLEDGEMENT

We would like to thank the Trustees of the British Museum for permission to include the photograph on page 9, and the Science Museum for permission to include the photograph on page 37.

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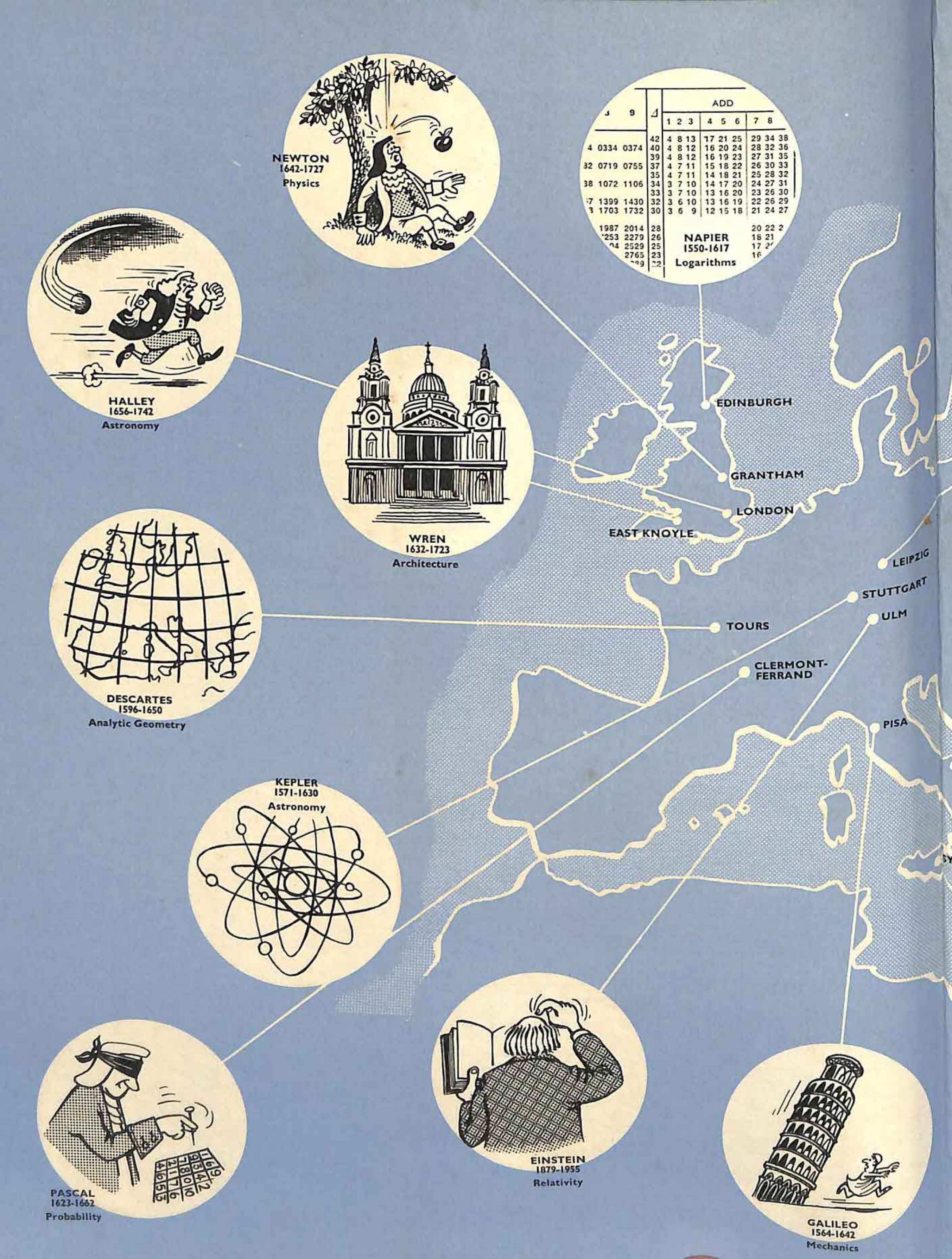
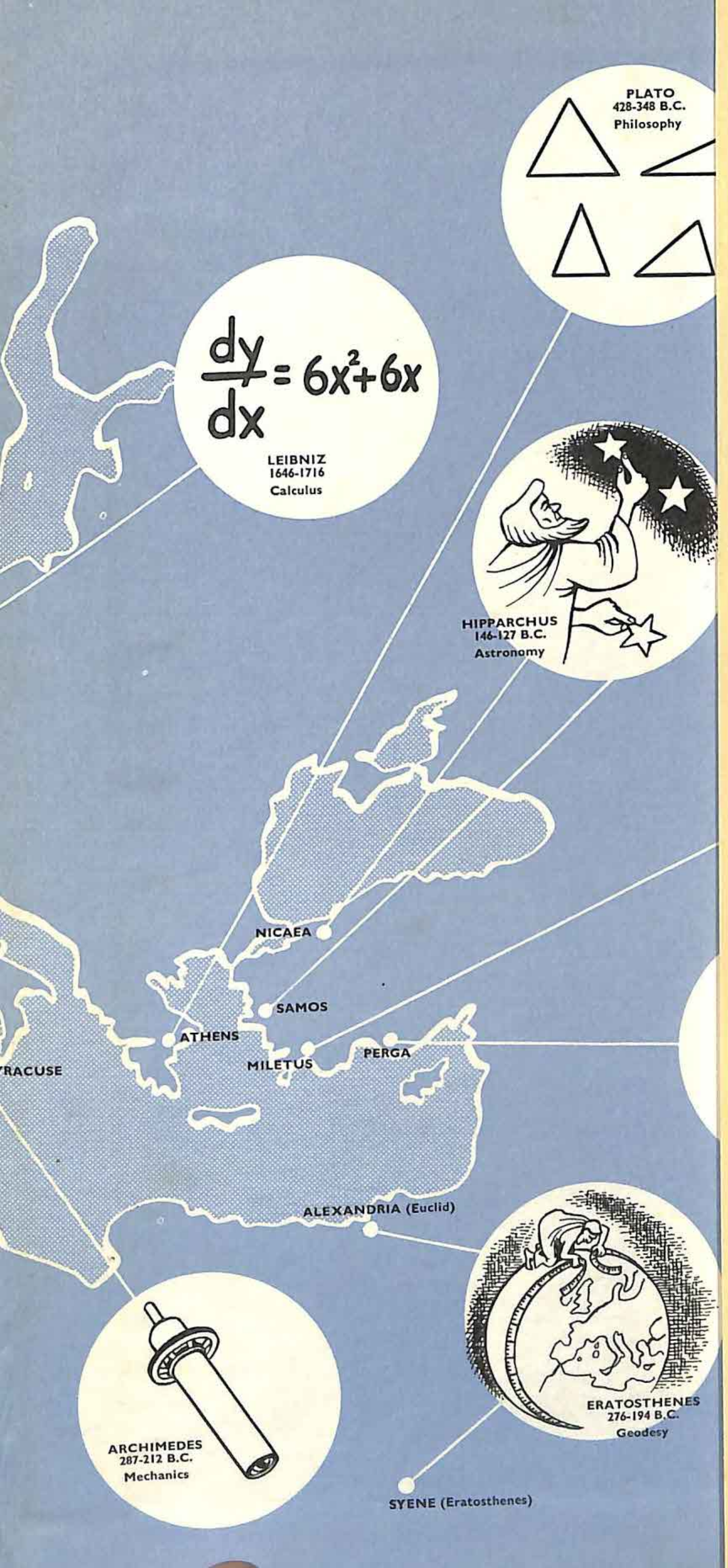

NEWTON
1642-1727
Physics
NAPIER
1550-1617
Logarithms
ADD
HALLEY
1656-1742
Astronomy
WREN
1632-1723
Architecture
DESCARTES
1596-1650
Analytic Geometry
KEPLER
1571-1630
Astronomy
PASCAL
1623-1662
Probability
EINSTEIN
1879-1955
Relativity
GALILEO
1564-1642
Mechanics
EDINBURGH
GRANTHAM
LONDON
EAST KNOYLE
LEIPZIG
STUTTGART
ULM
TOURS
CLERMONT-
FERRAND
PISA



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From the Beginning
From the Cave to the City
Man Makes his World
The Shape of the Earth
by Patrick Lynch

The Story of Ancient Egypt
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